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Optimal aggregate testing using Vandermonde polynomials and spectral methods

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Abstract

Recycled aggregate (RA) has been used in various construction applications around the world mainly as sub-grade, roadwork and unbound materials, but not in higher-grade applications. The major barrier encountered is the variation of quality within RA, which causes lower strength, and poorer quality. This work studies the relationships among six parameters describing the characteristics of RA: (i) particle size distribution, (ii) particle density, (iii) porosity and absorption, (iv) particle shape, (v) strength and toughness, and (vi) chemical composition. Samples of RA from 10 demolition sites were obtained with service life ranging from 10 to 40 years. One additional set of samples was specifically collected from the Tuen Mun Area 38 Recycling Plant. The characteristics of these eleven sets of samples were then compared with normal aggregate samples. A Vandermonde matrix for interpolation polynomial coefficient estimation is used to give detailed mathematical relationships among pairs of samples, which can be used to work out redundant tests. Different orders of interpolation polynomials are used for comparison, hence the best-fit equations with the lowest fitting errors from different orders of polynomials can be found. Fitting error distributions are then studied by using spectral methods such as power spectra and bispectra. From that, the best equations for result estimations can be obtained. This study reveals that there is strong correlation among test parameters, and by measuring two of them: either “particle density” or “porosity and absorption” or “particle shape” or “strength and toughness”, and “chemical content”, it is sufficient to study RA.

Keywords: Recycled aggregate; Concrete; Correlation; Property; Porosity; Strength; Construction

1. Introduction

Aggregate, in general, occupies about 70–80% of concrete volume and can therefore be expected to have important influence on concrete properties [1,2]. Its selection and proportioning should be given careful attention to control the quality of concrete structures. Apart from being used as an economical filler, aggregate generally gives concrete better dimensional stability and wear resistance. In choosing aggregate for a particular concrete, three general requirements should be considered: concrete economy, concrete strength, and concrete durability [2]. In addition, aggregate is more liable to deformation and less resistant than cement slurry due to their porosity [3]. As RA has higher porosity, it is more dependent on deformation and mechanically less resistant than the cement matrix coating after sufficient hardening time [3].

Rubble from demolished concrete building consists of fragments in which the aggregate is contaminated with hydrated cement paste, gypsum, and minor quantities of other substances. The size fraction that corresponds to fine aggregate mostly contains hydrated cement paste and gypsum and is unsuitable for making fresh concrete mixtures. However, the size fraction that corresponds to coarse aggregate, although coated with cement paste, has been successfully used in several laboratory and field studies [4]. A review of several studies indicated that compared with concrete containing natural aggregate, recycled aggregate concrete could have at least two thirds of the compressive strength and modulus of elasticity, hence meeting workability and durability standards [4]. A major

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Table 1
Assessment of the aggregate

Parameters	Tests
Particle size distribution	Test 1: 10 mm size aggregate of particle size distribution Test 2: 20 mm size aggregate of particle size distribution
Particle density	Test 3: 10 mm size aggregate of particle density on an oven-dried basis (in Mg/m^3) Test 4: 20 mm size aggregate of particle density on an oven-dried basis (in Mg/m^3) Test 5: 10 mm size aggregate of particle density on a saturated and surface dried basis (in Mg/m^3) Test 6: 20 mm size aggregate of particle density on a saturated and surface dried basis (in Mg/m^3) Test 7: 10 mm size aggregate of apparent particle density (in Mg/m^3) Test 8: 20 mm size aggregate of apparent particle density (in Mg/m^3)
Porosity and absorption	Test 9: 10 mm size aggregate of water absorption (in % of dry mass) Test 10: 10 mm size aggregate of saturated time for water absorption (in h) Test 11: 20 mm size aggregate of water absorption (in % of dry mass) Test 12: 20 mm size aggregate of saturated time for water absorption (in h) Test 13: 10 mm size aggregate of moisture content (in % of dry mass) Test 14: 20 mm size aggregate of moisture content (in % of dry mass)
Particle shape	Test 15: 10 mm size aggregate of flakiness index (in %) Test 16: 20 mm size aggregate of flakiness index (in %) Test 17: 10 mm size aggregate of elongation index (in %) Test 18: 20 mm size aggregate of elongation index (in %)
Strength and toughness	Test 19: 10% fine value (in kN) Test 20: aggregate impact value (in %)
Chemical composition	Test 21: 10 mm size aggregate of chloride content (in %) Test 22: 20 mm size aggregate of chloride content (in %) Test 23: sulphate content (in %)

obstacle in using rubble as aggregate for concrete is the cost of crushing, grading, dust controlling and separation of undesirable constituents. Crushed recycled concrete or waste concrete can be an economical aggregate source which is difficult to find, and is also important when waste disposal is increasingly becoming more costly [4]. This paper aims to study properties of aggregate; and to modify aggregate testing procedures by using Vandermonde polynomial interpolation and spectral methods.

2. Aggregate assessment

Aggregate quality is generally assessed by using 23 standard tests which are categorized into 6 parameters in this paper (Table 1): (i) particle size distribution; (ii) particle density; (iii) porosity and absorption; (iv) particle shape; (v) strength and toughness; and (vi) chemical composition.

The standard methods used for testing these aggregate properties are summarized in Table 2.

3. An interpolation process using Vandermonde matrix

Interpolation using polynomial fitting is a technique which uses polynomials of order up to 20 to fit a given set of data. This technique is well known because it is much better than the linear regression method of simply assigning the “line of best fit” to the data. Given a set of data in the form of $x(1), x(2), \ldots, x(N)$, with values of $y(1), y(2), \ldots, y(N)$, where N is the data length. The coefficients $c_1, c_2, \ldots, c_N$ of the interpolating polynomial which can be used to “best fit” the data relate the input x to the output y via the Vandermonde matrix of the form [5]:

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \ldots & x_0^{N-1} \\ 1 & x_1 & x_1^2 & \ldots & x_1^{N-1} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & x_N & x_N^2 & \ldots & x_N^{N-1} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \ldots \\ c_N \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \ldots \\ y_N \end{bmatrix}, \quad (1)$$

where the c matrix consists of coefficients of the polynomial. It should be stressed that the c matrix does not always exist; prompting that extra care must be taken when using the technique to interpolate different data sets.

Having obtained the c matrix, the interpolating polynomial is thus given by:

$$y_{\text{interpolate}} = c_N x^N + c^{N-1} x^{N-1} + \cdots + c_1 x + c_0, \quad (2)$$

Table 2
Standard used for aggregate

Properties of aggregate	Standard
Particle size distribution	
Sieve analysis	[11]
Particle density	
Particle density on oven-dried basis	[12]
Particle density on saturated and surface-dried basis	[12]
Apparent particle density	[12]
Porosity and absorption	
Water absorption	[12]
Moisture content	[25]
Particle shape	
Flakiness index	[15]
Elongation index	[16]
Strength and toughness	
Ten percent fine value (TFV)	[17]
Aggregate impact value (AIV)	[18]
Chemical composition	
Chloride content	[26]
Sulphate content	Manual of Alltech Ion Chromatography System ICM-300

which can be used to mathematically model the given data. It should also be noted that $y_{\text{interpolate}}$ generally resembles the shape of the fitted data. However, sometimes, it is difficult to find all coefficients for a particular data set. Thus, if the method is applicable to a set of data, then the process of studying and simulating the data becomes much easier and less time consuming as $y_{\text{interpolate}}$ can now be validly used. However, no numerical methods can completely simulate a given set of data, thus, there exists some marginal errors in curve fitting which generally do not significantly alter the results obtained by analysing $y_{\text{interpolate}}$. Even though interpolation and spectral techniques have been widely used in the field of signal and image processing [5], they have not been widely used in the field of construction material and management to process data and to study their correlation.

Out of the 23 tests, the first two tests do not have numerical results, leaving tests 3–23 applicable for interpolation. Every test from 3 to 23 is then used as an input with the other tests as outputs to obtain their mathematical relationships with the input test. For example, the first patch of interpolation uses test 3 as the input, thus tests 4–23 are used as the outputs. As a result, the relationships between test pairs 4 and 3, 5 and 3, 6 and 3 and so on until the last test pairs of 23 and 3 are obtained. In the second patch of interpolation, test 4 is used as the input and tests 5, 6, 7 until 23 as the outputs. The interpolation process continues until test 22 is taken as the input and test 23 as the output, in this case, there is only one pair of input and output in the interpolation patch. At the end of the whole interpolation process, by using one order, there are 210 equations describing the mathematical relationships among all the tests, i.e. every test is interpolated with every other test, and therefore it is not difficult to estimate the results of a particular test using one of the many equations obtained from the interpolation process. Ten different order polynomials are used to interpolate the data, yielding 2100 equations relating the results of all tests. The challenge is to choose the best polynomials with the lowest fitting errors. Fitting errors are estimated by taking the difference of the interpolation polynomial and the real data. Because there are 10 different polynomial orders, there exist 10 different mathematical equations which can be used to estimate results of a particular test 4. The same process is carried out for all tests and in all orders. It is clear that the more polynomials the interpolation process uses, the easier it is to simplify aggregate testing procedures as there is more than one equation relating results of a particular input to a particular output available for selection. The main difference of this paper and other papers is to use spectral methods such as power spectra and bispectrum to study the fitting errors instead of estimating the error's mean, thus revealing error uniformness and distribution. To choose satisfactory polynomials, the error upper limit is chosen to be 15% in this paper. It should be noted that interpolation equations possessing errors larger than the upper limit are considered to be invalid and hence cannot be used to estimate the results of the other tests.

The interpolating polynomials are generated by using the MATLAB package via the command *polyfit*. The order of the polynomials is considered to be an important parameter. For this particular set of data, polynomial orders of 1–10 are used to thoroughly study the effectiveness of the method. It should also be noted that the higher the order of the polynomial, the better the fit to the data. However, for data consisting of many abrupt changes, high-order polynomials cannot satisfactorily interpolate the data as will be shown later.

4. Spectral methods

4.1. The Fourier transform

The Fourier transform is a useful and powerful tool employed to study "frequency" components of signals and discrete data which are usually recorded in the time domain. After transforming the data into the frequency domain using the Fourier transform, the signal energy distribution at different frequencies is revealed. Effectively, the Fourier transform can be considered as a prism where white light can be split into its individual spectra. For the case of the Fourier transform, the signal energy is split over the signal's spectrum which consists of a number of frequencies at which harmonics and sub-harmonies are displayed. Mathematically, the Fourier transform $X(f)$ as a function of the frequency f is given as [6]:

$$X(f) = \int_{-\infty}^{+\infty} x(t)\,\mathrm{e}^{-j2\pi ft}\,\mathrm{d}t, \tag{3}$$

where $j^2 = -1$ is a complex constant, $\pi \approx 3.1415$ and $x(t)$ is the input signal or data. The input data or signal is usually a 1D array or 2D matrix.

To recover a time signal from its Fourier transform, the inverse Fourier transform is employed, which is mathematically given as:

$$x(t) = \int_{-\infty}^{+\infty} X(f)\,\mathrm{e}^{+j2\pi ft}\,\mathrm{d}f. \tag{4}$$

It should be noted that the Fourier transform is a complex number which is uniquely described by its magnitude and phase. Thus, it is clear that there are two ways of representing data: in the time domain and in the frequency domain using the Fourier transform. The transformation from time domain to frequency domain is achieved by using the operator $\mathrm{e}^{j\omega t}$, which can be given in the following equation as:

$$\mathrm{e}^{j\omega t} = \cos(\omega t) + j\sin(\omega t). \tag{5}$$

Frequency is normally defined as the number of repetitions over time and the concept of "frequency domain" is believed to be new in the field of construction material and management. Frequency is inversely proportional to time, which means the larger the time, the smaller the frequency and vice versa. Using the concept of frequency and time it can be said that data which have a long time span have densely concentrated spectra over a short frequency range and vice versa. The magnitude of the frequency components which are displayed over a frequency range or spectrum is defined as proportional to the signal energy. Signals which are continuous and periodic in time have densely concentrated energy spectra. For ease of understanding, the Fourier transform can be viewed as mapping of the energy distribution in the signal in the frequency domain at which harmonic peaks or dominant peaks represent the peak energy concentration in the waveform. For example, the Fourier transform of a constant signal which is continuous from $-\infty$ to $+\infty$ is an impulse whose energy concentration is theoretically perfect. A common and popular sinusoidal signal of frequency $f_0 = 1$ Hz has two impulses located at ± 1 Hz in its spectrum as shown in Fig. 1.

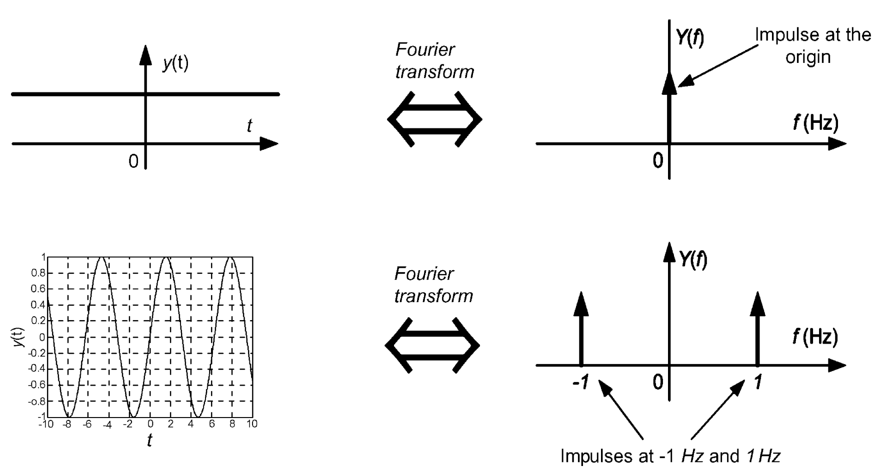

Fig. 1. The Fourier transforms of a constant straight line signal and a sinusoid $y(t) = \sin(t)$ using Eq. (1).

4.2. The power spectrum

The interpolation method is used to estimate the results of output tests from input tests. From that, it is possible to determine redundancy among the tests, in turn, significantly lowers the number of tests. To further study the correlation among the tests, spectral methods using the power spectrum and bispectrum are employed. The power spectrum $P(f)$ of a data set $x(t)$ is given in the following equation as:

$$P(f) = |X(f)|^2, \tag{6}$$

where $X(f)$ is the Fourier transform of the data or input signal. It is evident that the power spectrum is proportional to the square magnitude of the input signal's Fourier transform because the signal energy is directly related to its squared magnitude. It is important to stress that energy plays an important role in determining data characteristics, i.e. periodic, aperiodic or chaotic, detecting transitions from one state to another, i.e. from periodicity to chaos or periodicity to transient, and working out the energy weighting at different frequencies [6] which can be achieved by estimating the power spectrum of the input data. In the case of studying sample results of tests in construction material and management, the power spectrum is particularly useful as it can reveal the energy distribution of samples in each test. From that, the significance of each test can be assessed. In addition, the power spectrum can be used to classify different types of data including periodic, chaotic, transient and noise by interpreting its shapes and frequency range [7]. Recently, the power spectral method has been successfully used to identify dominant criteria [8] in environmental surveys by studying their energy distribution. Moreover, as data processing and analyses are increasingly important, this further strengthens the idea of using spectral methods in the field of construction material and management. The only drawback of the power spectrum is that its phase information is suppressed which means that two different data sets could have identical power spectra. To overcome this problem and to further study the correlation among the tests and samples, the bispectral method is employed.

4.3. The bispectrum

To further study the data, a bispectral method is introduced which shows the correlation among the tests at various "frequencies". The bispectrum $B(f_1, f_2)$ has been widely employed in the field of high-order statistics to study data correlation in 3D and is given by [9]:

$$B(f_1, f_2) = X(f_1)X(f_2)X^*(f_1 + f_2), \tag{7}$$

where the symbol "*" means complex conjugate.

It is clear that the bispectrum is strongly dependent on the Fourier transform of the input signal. From Eq. (7), the term $X^*(f_1 + f_2)$ represents the correlation among various frequency terms in the $(f_1 + f_2)$ plane. To estimate the bispectrum, the mean value of the data is removed to eliminate sudden spikes and pulses which could lead to misleading interpretation. In MATLAB, this can be done by using a *detrend* $(\cdot)$ function. After that, the data are windowed using a Hanning window via the command *hanning* $(\cdot)$ provided in MATLAB. In addition, the data are also normalised by dividing each column by its largest item so that abrupt changes are nullified. The Fourier transforms of the detrended data are then calculated, in this case, there are 21 out of 23 tests having numerical results, yielding 21 Fourier transforms. In this paper, the bispectrum of an error matrix of 210×10 is calculated to show correlation among the fitting errors and also error uniformness.

Unlike the power spectrum which suppresses the phase information in the data, the bispectrum uniquely gives the phase information, i.e. the correlation among a number of frequencies, which enable detailed studied on correlation among the tests. However, because the phase information is usually difficult to interpret, the magnitude bispectrum is usually employed as the main tool for data analyses.

5. The study

Ten series of RA samples (samples 1–10) were obtained from 10 demolition sites with service life ranging from 10 to 40 years. Sample 11 was specifically collected from the Tuen Mun Area 38 Recycling Plant. Samples one to eleven are then compared with normal aggregate which is sample 12. The results of 23 tests for samples 1–12 are summarized in Table 3.

5.1. Particle size distribution

Since the strength of fully compacted concrete with a given water/cement ratio is independent of the particle size distribution (sieve analysis of the aggregate), sieve analysis is important only if it affects fresh concrete workability [10]. Samples 1–12 have met the particle size distribution criterion of being 10 and 12 mm single-size aggregate as stated in BS 882 [11] (see Table 3).

Table 3
Summary of results from samples 1–12

Sample	Particle size distribution		Particle density						Porosity and Absorption					
	Sieve analysis		Particle density on an oven-dried basis (Mg/m^3)		Particle density on a saturated and surface-dried basis (Mg/m^3)		Apparent particle density (Mg/m^3)		Water absorption (as % of dry mass)				Moisture content	
	10 mm	20 mm	10 mm	20 mm	10 mm	20 mm	10 mm	20 mm	10 mm		20 mm		10 mm	20 mm
									Water absorption	Saturated time[a] (in h)	Water absorption	Saturated time[a] (in h)		
1	Pass	Pass	2.16	2.20	2.33	2.36	2.60	2.61	5.83	96	6.89	91	1.01	1.24
2	Pass	Pass	2.22	2.14	2.38	2.32	2.64	2.61	6.36	85	6.40	82	1.05	1.21
3	Pass	Pass	2.20	2.18	2.36	2.35	2.62	2.63	7.50	115	7.35	122	1.35	1.35
4	Pass	Pass	2.20	2.20	2.37	2.36	2.65	2.63	6.93	120	7.25	120	1.25	1.35
5	Pass	Pass	2.15	2.19	2.32	2.34	2.59	2.59	7.31	124	6.82	108	1.25	1.25
6	Pass	Pass	2.25	2.27	2.41	2.42	2.66	2.67	5.20	117	5.77	116	0.98	0.97
7	Pass	Pass	2.11	2.13	2.31	2.31	2.61	2.60	8.74	120	7.30	122	1.02	0.95
8	Pass	Pass	2.10	2.12	2.30	2.31	2.62	2.61	8.58	119	7.99	102	1.63	1.35
9	Pass	Pass	2.21	2.24	2.37	2.39	2.64	2.63	6.94	127	6.11	118	1.24	0.84
10	Pass	Pass	2.20	2.23	2.36	2.36	2.60	2.57	6.85	100	5.95	77	1.26	1.42
11	Pass	Pass	2.46	2.53	2.53	2.58	2.65	2.66	2.63	24	1.65	24	0.49	0.33
12	Pass	Pass	2.59	2.62	2.62	2.64	2.67	2.66	0.77	24	0.57	24	0.15	0.15

Sample	Particle shape				Strength and toughness		Chemical composition		
	Flakiness index (%)		Elongation index (%)		TFV (kN)	AIV (%)	Chloride content (%)		Sulphate content (%)
	10 mm	20 mm	10 mm	20 mm			10 mm	20 mm	
1	11.13	9.68	29.00	16.19	93.89	33	0.0078	0.0089	0.031
2	10.44	10.08	24.18	25.15	61.36	36	0.0108	0.0091	0.017
3	15.17	8.61	20.99	22.78	107.42	31	0.0013	0.0019	0.005
4	15.42	7.91	27.29	24.05	112.82	23	0.0019	0.0019	0.005
5	17.82	12.96	36.13	23.86	92.09	32	0.0054	0.0061	0.006
6	11.96	9.93	26.43	21.91	155.53	25	0.0008	0.0025	0.006
7	12.86	5.70	21.56	22.18	110.18	30	0.0976	0.0902	0.013
8	15.12	9.78	27.41	28.26	83.48	34	0.0013	0.0014	0.005
9	13.78	12.17	21.92	18.25	92.87	36	0.0459	0.0352	0.024
10	16.47	9.92	30.46	21.09	89.91	28	0.0494	0.0430	0.018
11	25.97	29.52	34.62	33.76	102.97	33	0.0021	0.0070	0.008
12	28.27	22.52	28.20	26.01	189.38	21	0.0012	0.0016	0.003

[a] The saturated time is obtained when the mass becomes steady.

5.2. Particle density

The particle density of aggregate is the ratio of the mass of a given volume of material to the mass of the same volume of water [12]. Aggregate particle density usually is an essential property for concrete mix design and also for calculating the volume of concrete produced from a certain mass of materials [13]. As the density of cement mortar (around 1.0–1.6 Mg/m^3) is less than that of stone particles of about 2.60 Mg/m^3 [14], the smaller the particle density, the higher the cement mortar content adhering to the RA. The average results of the three different tests based on oven-dried basis, saturated and surface-dried basis, and apparent particle density, were measured and are presented in Table 3.

From Table 3, samples seven and eight have the lowest values of particle density, inferring the highest amount of cement mortar adhering to RA, while sample 12 (normal aggregate) has the highest particle density. Furthermore, particle densities of 20 mm aggregate are larger than those of 10 mm aggregate, inferring a higher amount of cement mortar attached to the 10 mm aggregate. This also implies that the larger the aggregate size, the smaller the amount of cement mortar attached to its surface, yielding better aggregate quality.

Polynomial fitting of tests (outputs) based on the results of a particular test (input) can be achieved by using an appropriate polynomial order. Generally, the higher the polynomial order, the better the fitting. However, it is not always the case if there are abrupt changes in the outputs because a very high-order polynomial is required, which is not practical if the order is larger than the upper limit of 20 given in MATLAB. Thus, care must be taken to choose the appropriate order for the interpolation polynomial, otherwise large errors can be generated. The fitting errors of all orders are given to assess the effectiveness and validity of each order (see Appendix B).

Eqs. (8)–(217) mathematically describe the relationship among the tests and are given in Appendix A. Eqs. (8)–(112) give the relationships of tests 4–23 which are considered as the outputs using the best-fit polynomials. Simulation results show that the errors for tests 3–20 are mostly acceptable with the maximum errors lower than the chosen error limit of 15%.

5.3. Porosity and absorption

The overall porosity or absorption of aggregate either depends on a consistent degree of particle porosity or represents an average value for a mixture of variously high and low absorption materials [13]. In this study, both the rate of water absorption and moisture content are used to assess the level of porosity and absorption of the samples.

The water absorption and moisture content of recycled aggregate (samples 1–12) are generally higher than that of normal aggregate (sample 12) (see Table 3). Ten millimetre size aggregate of sample 7 exhibits the highest water absorption rate and moisture content of about 9.06 and 1.70, respectively, and 20 mm aggregate from sample 12 has the lowest water absorption rate and moisture content of about 0.53 and 0.15, respectively. One of the most obvious attributes between RA and normal aggregate is the higher water absorption rate and moisture content, which are affected by the amount of cement paste sticking on the aggregate surface. Cement mortar describes the soundness of aggregate since its porosity is higher than that of aggregate, i.e. RA with a higher absorption rate tends to be worsened in strength and resistance under freezing and thawing conditions [18–20] than aggregate with a lower absorption rate. In most samples, the water absorption rate of 20 mm aggregate is less than that of 10 mm aggregate, inferring that larger size aggregate may have less cement mortar adhered to its surface, leading to a lower water absorption rate as explained in the last section.

Using a standard testing method [12] of waiting for 24 h before measuring water absorption is not appropriate for recycled aggregate due to the high amount of loosely bonded cement paste on particles resulted from the crushing process. Experiments showed that the required time to fully saturate RA depends on its quality which can be determined by the amount of cement paste adhering on its surface. In most cases, the required time is more than 24 h. From experiment, it is believed that full saturation can take up to 48 h; some may take 72 h or even 120 h. Thus, a fixed duration of 24 h set by BS 812: Part 2 [12] may not be sufficient for RA. Relationships of tests 10–23 as the outputs based on tests 9–14 as the inputs are described by Eqs. (113)–(181) using the best-fit polynomials.

5.4. Particle shape

The characteristics and variations of the shape of aggregate particles can affect concrete strength and workability [13]. The shape of aggregate particles is best described by using two principal parameters: ‘sphericity’ and ‘roundness’. Aggregate particles are classified as flaky when they have a thickness (smaller dimension) of less than 0.6 of their mean sieve size. For example, a mean sieve size of 7.5 mm is the mean of two successive sieves at 5 and 10 mm [15]. Aggregate particles are classified as elongated when they have a length (greatest dimension) of more than 1.8 of their mean sieve size [16].

BS 882 [11] now provides limits for flakiness (particle thickness relative to other dimensions). Such aggregate particles could lead to either water gain under the aggregate, causing planes of weakness, or higher water demand and lower strength in concrete. BS 882 [11] limits the flakiness index determined in accordance with BS 812: Part 105:1 [15] to about 50% for uncrushed gravel and 40% for crushed rock or crushed gravel, with a warning that lower values may have to be specified for special circumstances such as pavement wearing surfaces. All the 12 samples in this study have a flakiness index lower than 40%. Mathematical relationships of tests 17–23 as the outputs based on tests 15–18 as the inputs are given in Eqs. (182)–(207) by using the best-fit polynomials.

5.5. Strength and toughness

It is important that aggregate used for concrete be 'strong' in a general sense [14]. In most cases, inherent aggregate strength is dependent upon aggregate 'toughness', a property broadly analogous to 'impact strength'. In this study, 10% fine values (TFV) and aggregate impact values (AIV) are used to determine the strength and toughness of the 12 samples.

The TFV measures the resistance of aggregate to crushing which is applicable to both weak and strong aggregates [17], the larger the TFV value, the more resistant the aggregate to crushing [13]. The AIV relatively measures the resistance of aggregate to sudden shock or impact, which in some aggregate is different from its resistance to a slowly applied compressive load [18]. The smaller the AIV value, the tougher the aggregate or more impact resistant than higher strength concrete aggregate [13]. Out of the 12 samples, sample 12 (ordinary aggregate) has the highest value of TFV and the lowest value of AIV at 189 kN and 21%, respectively; while sample 2 achieves the lowest value of TFV and the highest value of AIV at 61 kN and 36%, respectively (see Table 3). The obvious reason is that the cement paste attached to the RA directly affects its strength.

BS 882 [11] provides limits for TFV and AIV, minimum of 150 kN and 45%, respectively, according to the type of concrete in which the aggregate is used. According to the British Standard, samples 6 and 12 can be used for structural elements, samples 4 and 7 for pavement work and other samples confined to non-structural elements. The mathematical relationships of tests 20–23 as the outputs based on tests 19 and 20 as the inputs are given in Eqs. (208)–(214) by using the best-fit polynomials.

5.6. Chemical composition

Chloride and sulphate contents of RA are critical. Chloride contamination of recycled aggregate mainly derived from marine structures or similarly exposed structural elements is of concern which can lead to corrosion of steel reinforcement. However, for most RA (samples 1–6 and 8–12), the chloride ion contents are low and within the limit of standards (under 0.05%). Nevertheless, sample 7 falls beyond the limit with chloride contents of about 0.0976% and 0.0902% for 10 and 20 mm aggregates, respectively (see Table 3). From further investigation of the RA of sample 7, some shell (from fine marine aggregate) contents were found. The major reason may be the use of marine water or stream water for concrete mixing during periods of shortage of fresh water supply in the 1960s, which has been banned since 1970s. This could have increased the chloride composition in the sample.

In general, RA has a higher sulphate content than natural aggregate. The occurrence of sulphate-based products such as plaster as contaminants in demolition waste is common. Consideration must be given to the use of sulphate resisting cement in situations where plaster contamination is suspected [19]. However, gypsum plaster is rarely used in Hong Kong where lime plaster is more common. In fact, the highest recorded sulphate content is about 0.0308% for sample 1, which is still within the standard of 1% (see Table 3). Therefore, contamination of sulphate content is not a major problem for RA in Hong Kong. The mathematical relationships of tests 22 and 23 as the outputs with tests 21 and 22 as the inputs are given in Eqs. (215)–(217) by using the best-fit polynomials.

Using the results obtained in Sections 5.1–5.6, the best-fit polynomials are shown in Eqs. (8)–(217). From the results obtained in this paper, the tests can be divided into two major groups: group one consists of tests 3–20, and group two consists of tests 21–23. It is clear that the tests in group one are strongly correlated which as seen in Eqs. (8)–(214). This means that the results of any test in this group can be successfully estimated by using the results of another test from the same group. The error percentage of the first test group is satisfactory. However, there are a small number of tests possessing errors of more than 15%, which do not affect the findings in this paper since there is more than one mathematical expression describing them.

It should also be noted that there are some satisfactory relationships among the three tests in the second test group (tests 21–23). However, most equations in this group possess high error percentage which suggests that they are poorly correlated. It can be suggested not to use Eqs. (215)–(217) to predict the results of tests in the first test group to estimate the results of the second test group. As a result, only two dominant tests out of tests 3–23 are required instead of 21 tests being routinely conducted in total in the industry. In addition to tests 1 and 2, there are four tests which are required to be conducted in total. It should also be noted that out of tests 3–20, the results of only one of these tests is required which provides flexibility in conducting the tests depending on the conditions and equipment availability. As construction sites in Hong Kong are limited in size, eliminating redundant tests significantly lowers cost and shortens aggregate testing time, yielding more efficient space usage on site and many other benefits for the construction industry. Table 4 summarises the findings of the paper.

To assess the effectiveness of the interpolation process using different orders, fitting errors of interpolation polynomials of orders 1–10 are estimated and given in Fig. 2. Fitting errors are the difference between the real data and values of the corresponding polynomials. It is clear that the smaller the fitting error, the better the polynomial fitting. The maximum allowable fitting error is chosen to be 15% in this paper. In addition, by using spectral methods, it is possible to study fitting error distribution and uniformness, which can be used to study error behaviour, i.e. predict error magnitude for different tests.

Figs. 3 and 4 plot the normalised and absolute errors of all orders, respectively. It should be noted that the normalised errors of all orders are plotted for comparison purposes only. The absolute errors plotted in Fig. 4 give more insight to the effectiveness of

Table 4
Comparison of the normal testing method and the new testing method using the Vandermonde interpolation technique

Test number	Normal method	Vandermonde interpolation technique
Tests 1 and 2	Conducted both. Not applicable to the interpolation process since these tests do not have numerical results	
Tests 3–20	Conducted all	Conducted 1 out of 18 tests
Tests 21–23	Conducted all	Conducted 1 out of 3 tests
Total number of required tests	21	2

orders one to eight. Orders 9 and 10 are not included in Fig. 4 because their absolute fitting errors are much larger than those of the lower orders, making it impossible to plot them on the same scale. To further study the fitting errors, the bispectrum of the error matrix of all orders is computed and plotted in Fig. 6. The power spectra of the fitting errors of each order are also plotted in Fig. 7.

From Figs. 3 and 4, it is clear that the fitting errors of all orders vary uniformly among the tests. It should also be clear that for orders one to seven, the errors are more uniformly distributed than those of orders 8–10, suggesting that high-order polynomials are not suitable for modeling the data in this case. This is evidently reflected by having large "harmonic" spikes as shown in Fig. 3. Fig. 5 gives a useful plot of average fitting errors using all polynomial orders in which it is clear that orders seven and eight yield

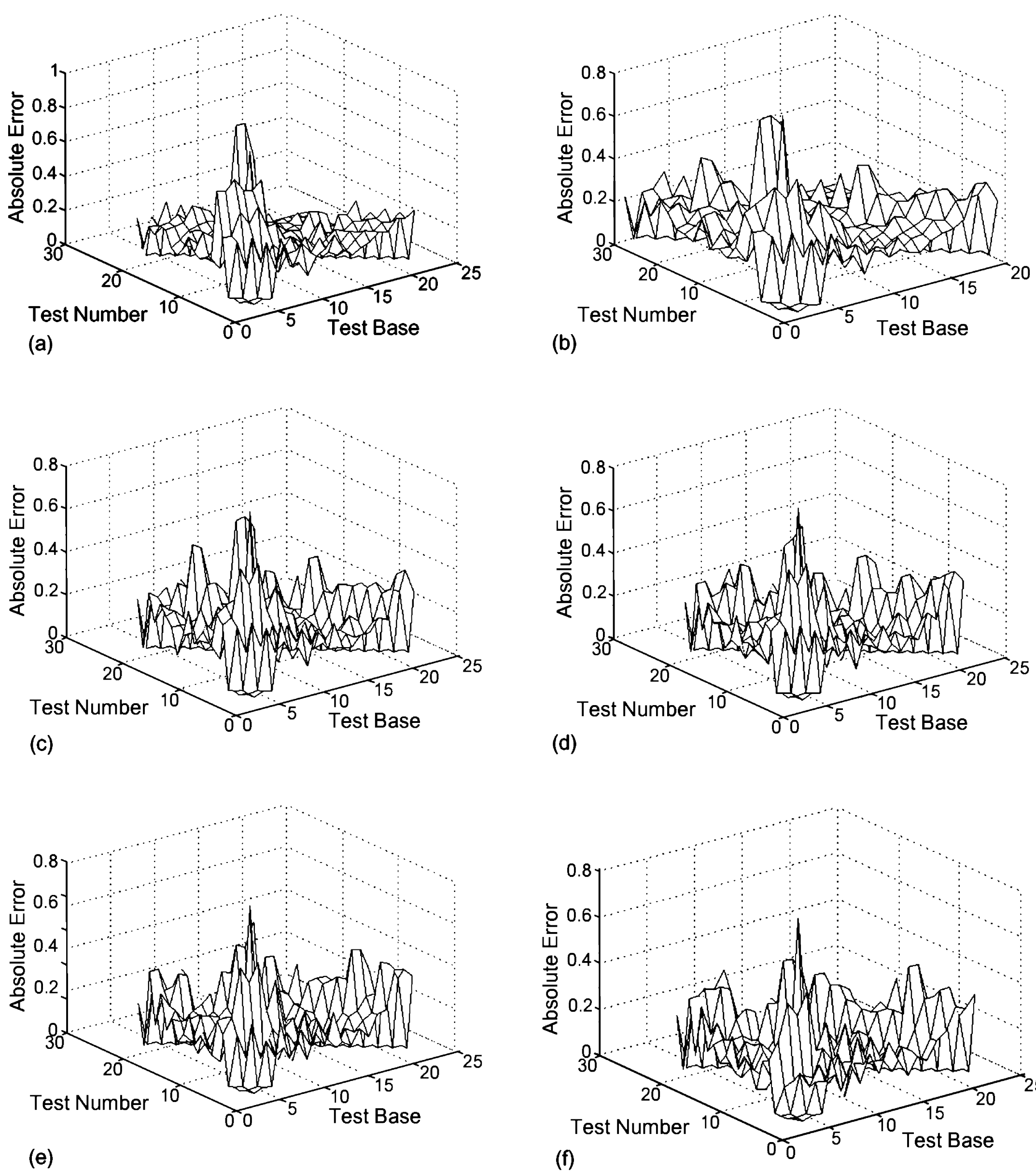

Fig. 2. Mesh plots of the error matricies of the (a) 1st-order polynomials; (b) 2nd-order polynomials; (c) 3rd-order polynomials; (d) 4th-order polynomials; (e) 5th-order polynomials; (f) 6th-order polynomials; (g) 7th-order polynomials; (h) 8th-order polynomials; (i) 9th-order polynomials; (j) 10th-order polynomials.

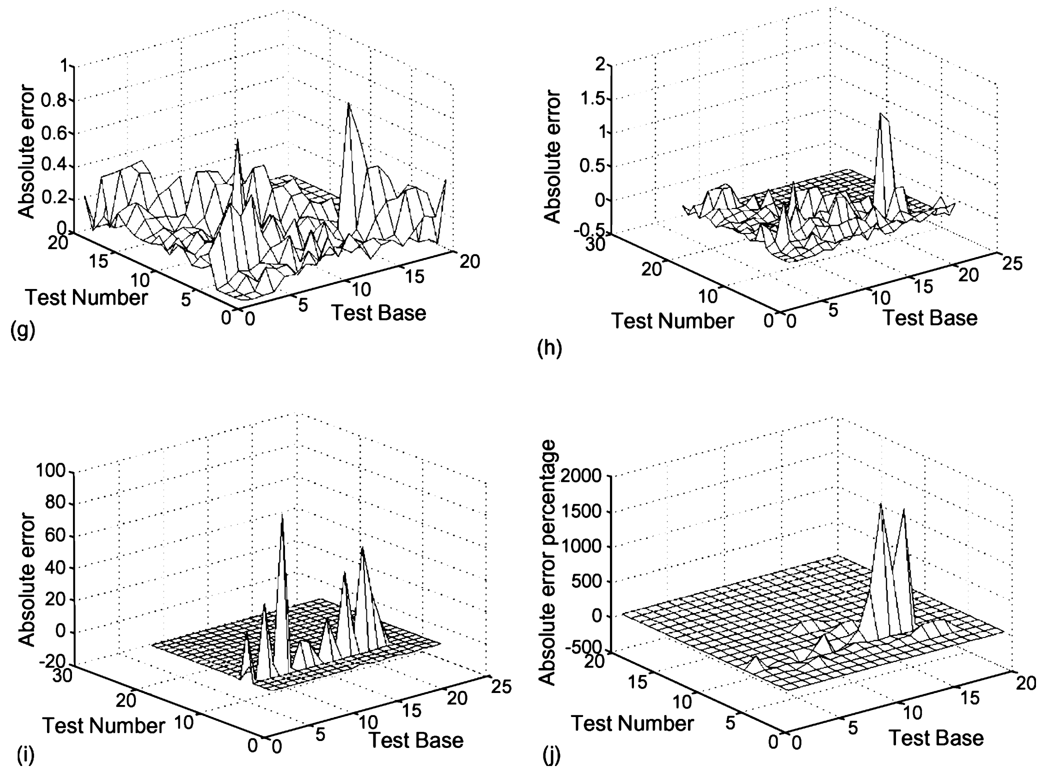

Fig. 2. (*Continued*).

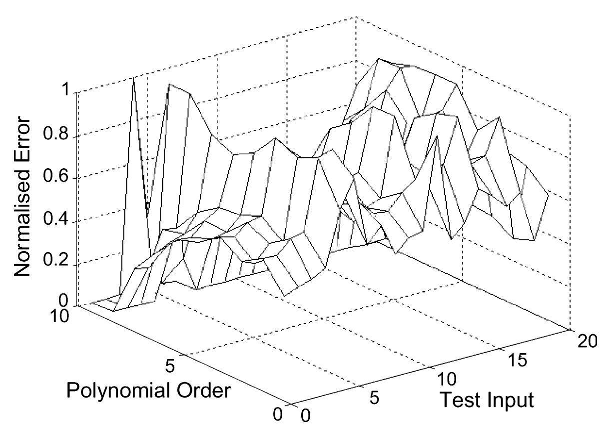

Fig. 3. Mesh plot of the normalised average error matrix of all test inputs using orders 1–10. This graph is given for comparison purposes only as the 10th-order polynomials possess larger errors compared with the other polynomials.

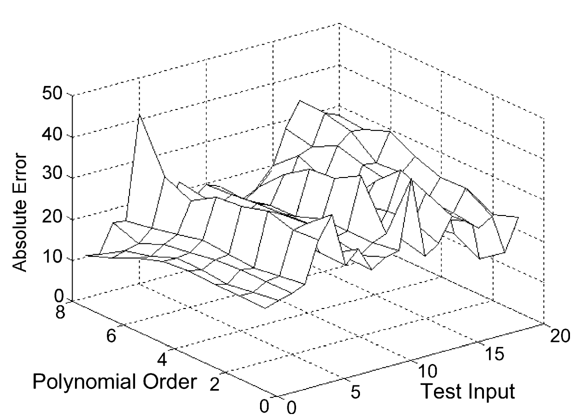

Fig. 4. Mesh plot of the *absolute* average error matrix of all test inputs with orders one to eight.

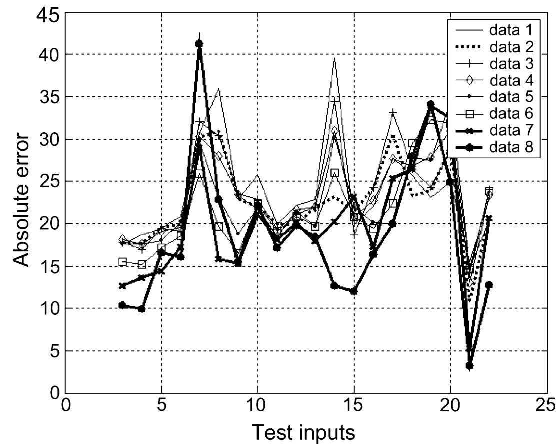

Fig. 5. Absolute average errors of all test inputs with orders one to eight.

the smallest errors. In addition, it is also clear that the eighth-order polynomial possesses smaller fitting errors than those of the seventh-order, suggesting that the former can be a better choice to model the data. However, one major advantage of the seventh-order polynomial over the eighth-order polynomial is that the errors of the former are more uniformly distributed. Thus, by using the seventh-order polynomial for the interpolation process, it is possible to predict the fitting errors of different test inputs. Compared with the seventh-order, the eighth-order polynomial possesses smaller fitting errors at test inputs 3, 4, 6, 14, 15, 17, 21 and 22, and larger errors at test inputs 7, 8 (apparent particle density), 10 (time period of water absorption), 13 (moisture contents of 10 mm aggregate) and 19 (10% fine value) in which for test input 7, there is a sudden jump in its fitting error which suggests that the eighth-order is more unstable and unpredictable. Further, it gives higher fitting errors for test input 19 which is an important test which should be accurately modeled. However, it should not be forgotten that the eighth-order polynomial does give smaller fitting errors at some other test inputs suggesting that it is also a useful polynomial for the interpolation process. Thus, there exists a trade-off between unpredictability and error magnitude at some particular important tests. By considering all aspects of the seventh- and eighth-order polynomials, it can be suggested that the seventh-order is more suitable for the interpolation process for this data set. Apart from the seventh- and eighth-order polynomials, other smaller orders give fine results but with larger average errors. The 9th- and 10th-order polynomials possess larger fitting errors and thus they should not be employed for the interpolation process in this case. At this point, the answer to the question raised in Section 3 is due to the large fitting error generated by orders 9 and 10. It is clear that the larger the order, the larger the error because large-order polynomials possess sharp edges and spikes which are not present in the data, causing large fitting errors. Thus, orders seven or below should be used for the interpolation process to study the data.

From Fig. 6, it is clear that the fitting errors of polynomials of orders less than five are strongly correlated among all equations. Further, it is clear that these fitting errors are not uniformly distributed. In addition, because the bispectrum's magnitude is non-zero, it is clear that these orders possess larger errors than those of orders 5–10. It should be noted that the number of equations displayed in Fig. 6 is only 105 as the other half of the bispectrum is identical. For polynomials of orders 5–10, there is much less correlation in the fitting errors for these polynomials suggesting that better estimation results can be obtained compared to those using the first

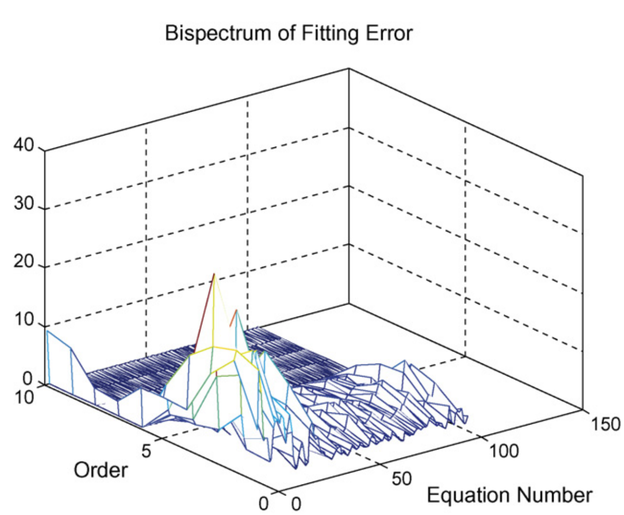

Fig. 6. Mesh plot of bispectrum of the error matrix of all orders.

five orders. The bispectral magnitude corresponding to these orders is also much smaller than that of the first five orders suggesting that their fitting errors are much smaller. One major drawback of the higher-order polynomials is that the very first few equations do not possess low fitting errors.

It is clear that the bispectrum is a useful tool to assess the error distribution and correlation, however, to assess error uniformness, the power spectrum should also be used. Fig. 7 shows the power spectra of fitting errors of orders 1–10 in which distinctive spikes are clearly displayed suggesting periodic characteristics in the fitting errors of most orders. This feature is also strongly revealed by examining the bispectrum given in Fig. 6 for the first five orders. Out of the last five orders, orders 9 and 10 possess smooth power spectra as can be seen in Fig. 7(i) and (j). It should be noted that the spikes represent "dominant harmonics" in the errors, which means they are periodic as can be shown in Fig. 1 for the power spectrum of a sinusoid consisting of two distinctive harmonic spikes. By having smooth power spectra, it can be suggested that the errors of orders 9 and 10 are not periodic, but tend to be random or chaotic or in a transition to chaotic [20–24]. It is also clear that even though these orders yield small fitting errors, usually, they are unpredictably large which explains their chaotic and random nature. For orders five to eight, it is clear that their fitting errors are more periodic and more predictable. The error uniformness is determined by the number of distinctive spikes in the power spectrum,

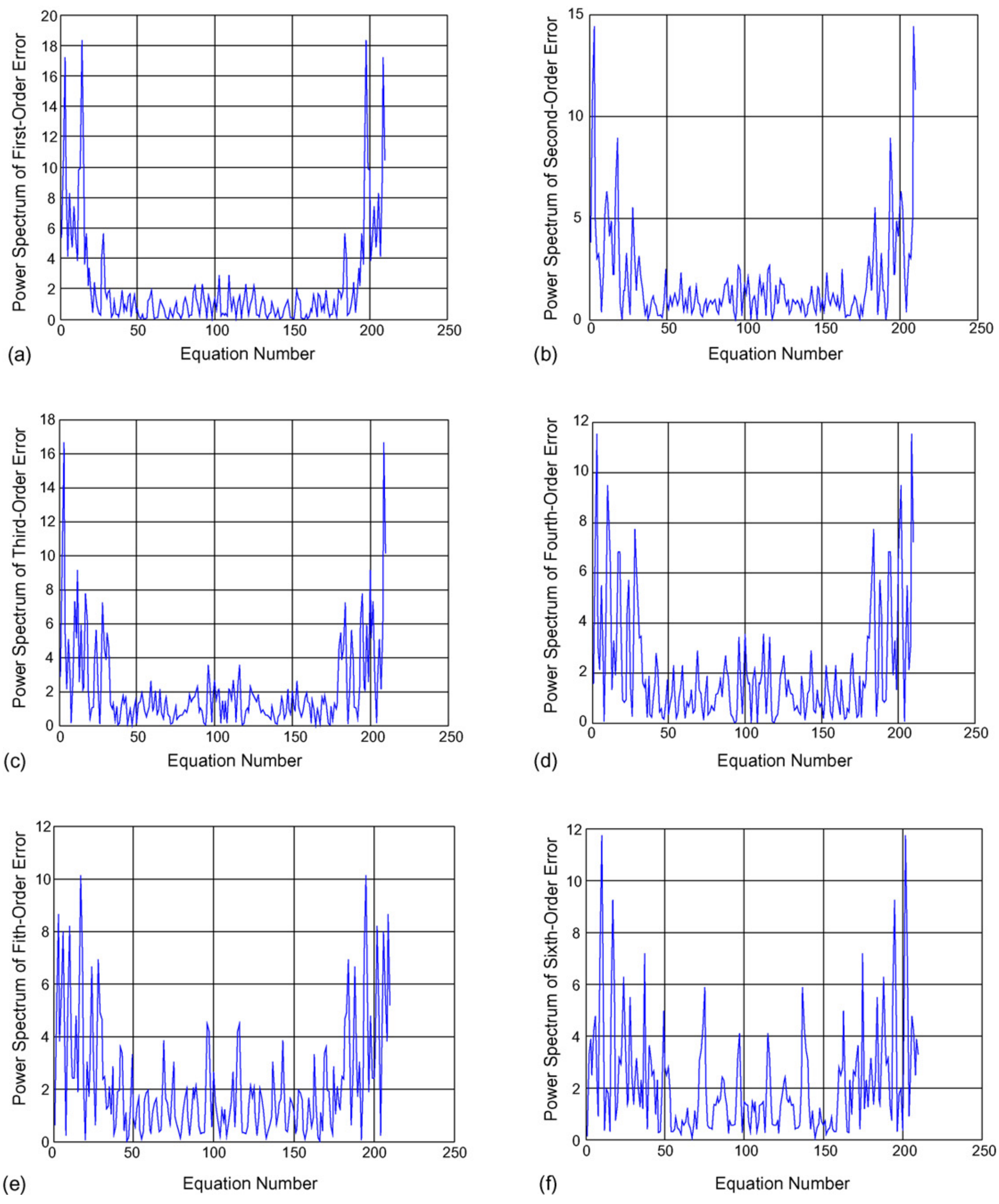

Fig. 7. The power spectra of the (a) 1st-order polynomials; (b) 2nd-order polynomials; (c) 3rd-order polynomials; (d) 4th-order polynomials; (e) 5th-order polynomials; (f) 6th-order polynomials; (g) 7th-order polynomials; (h) 8th-order polynomials; (i) 9th-order polynomials; (j) 10th-order polynomials.

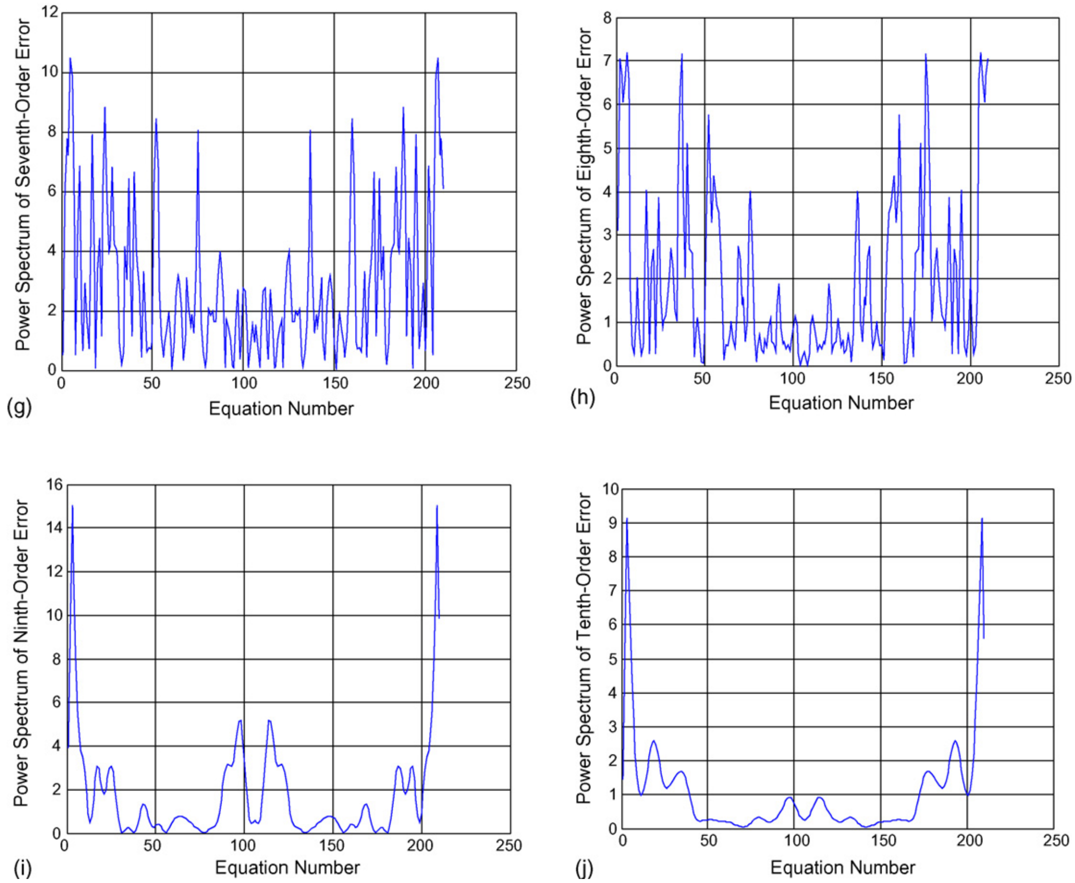

Fig. 7. (*Continued*).

i.e. the more spikes, the more uniform the error. In this case, orders seven and eight possess the most uniform errors even though their error magnitude may not be smallest. This provides predictability and uniformness in the errors which are desirable features in aggregate testing, because the easier to predict the error, the better the approximation method. It is also clear that the findings obtained by using the spectral methods are consistent with the findings obtained by using the normal average error calculations as shown in Figs. 2–5. It should be stressed that the spectral methods are the only methods which can show error correlation and uniformness. Thus, they are considered to be effective and powerful tools for data analysis in the field of construction material and management, especially for optimal aggregate testing.

As has been shown in this paper, polynomial interpolation and spectral methods are useful tools which can find a wide range of applications in the construction industry. Possible further applications using Vandermonde polynomial interpolation and spectral methods include identifying dominant criteria in affecting the environmental performance; correlating quality prevention and failure factors in construction industry; predicting the behaviour of unknown data to see when and where the transition from periodicity to chaos is in construction management and material engineering; and studying the relationship between concrete bahviour and its characteristics.

6. Conclusions

To have wide adoption of RA, it is essential to carefully assess its properties including particle density, porosity and absorption, particle shape, strength and toughness, and chemical composition. Sieve analysis should also be done to make good concrete proportioning. It has been found that all six parameters have direct relationship with the cement mortar adhering on the surface of aggregate leading to lower particle density, higher water absorption and lower 10% fine value. The RA from sample 7 exhibits the lowest quality because marine or stream water has been used in concrete mixing, which, however, is still adoptable for non-structural construction applications. Further, it has been found that there is strong correlation among some of the parameters which can be used to simplify aggregate testing processes. For example, by measuring one of “particle density”, “porosity and absorption”, “particle shape” and “strength and toughness” and “chemical content”, it is sufficient to assess the characteristic and properties of RA. A new technique of using interpolation polynomials of orders 1–10 has been employed in this paper. New spectral methods using the power spectrum and bispectrum have been introduced in this paper to study error correlation and uniformness. Fitting errors of interpolation polynomials have been estimated in which it was shown that polynomials of orders one to eight yield satisfactory results by providing periodic and predictable fitting errors of small magnitude. Orders 9 and 10 have been shown to possess random or chaotic fitting errors suggesting that they are not suitable for modeling the collected data presented in this paper. Out of the 10 orders, order 7 is the optimum order for use with the interpolation technique to process the data. This paper has shown that interpolation techniques can be successfully used to process data in the field of construction material and management.

Appendix A. Best-fit polynomials for assessing aggregate characteristics

$$y_4 = 10^{15}(0.0040x_3^9 - 0.0308x_3^8 + 0.1062x_3^7 - 0.2132x_3^6 + 0.2750x_3^5 - 0.2363x_3^4 + 0.1353x_3^3 - 0.0498x_3^2 + 0.0107x_3 - 0.0010 \quad \text{(with error of 0.88\%)} \tag{8}$$

$$y_5 = 10^{13}(-0.0003x_3^8 + 0.0023x_3^7 - 0.0069x_3^6 + 0.0120x_3^5 - 0.0131x_3^4 + 0.0091x_3^3 - 0.0040x_3^2 + 0.0010x_3 - 0.0001 \quad \text{(with error of 0.12\%)} \tag{9}$$

$$y_6 = 10^{13}(0.0014x_3^8 - 0.0096x_3^7 + 0.0292x_3^6 - 0.0508x_3^5 + 0.0552x_3^4 - 0.0383x_3^3 + 0.0166x_3^2 - 0.0041x_3 + 0.0004 \quad \text{(with error of 0.60\%)} \tag{10}$$

$$y_7 = 10^{15}(0.0006x_3^9 - 0.0045x_3^8 + 0.0156x_3^7 - 0.0315x_3^6 + 0.0407x_3^5 - 0.0350x_3^4 - 0.0201x_3^3 - 0.0074x_3^2 + 0.0016x_3 - 0.0002 \quad \text{(with error of 0.42\%)} \tag{11}$$

$$y_8 = 10^{15}(0.0050x_3^{10} - 0.0361x_3^9 + 0.1113x_3^8 - 0.1887x_3^7 + 0.1821x_3^6 - 0.0813x_3^5 - 0.0192x_3^4 + 0.0479x_3^3 - 0.0280x_3^2 + 0.0078x_3 - 0.0009) \quad \text{(with error of 2.18\%)} \tag{12}$$

$$y_9 = 10^{15}(0.0069x_3^{10} - 0.0362x_3^9 + 0.0489x_3^8 + 0.1012x_3^7 - 0.4791x_3^6 + 0.8371x_3^5 - 0.8491x_3^4 + 0.5413x_3^3 - 0.2150x_3^2 + 0.0489x_3 - 0.0049) \quad \text{(with error of 2.13\%)} \tag{13}$$

$$y_{10} = 10^{15}(0.0254x_3^9 - 0.1964x_3^8 + 0.6746x_3^7 - 1.3509x_3^6 + 1.7377x_3^5 - 1.4890x_3^4 + 0.8499x_3^3 - 0.3116x_3^2 + 0.0666x_3 - 0.0063 \quad \text{(with error of 5.22\%)} \tag{14}$$

$$y_{11} = 10^{11}(0.0327x_3^4 - 0.1157x_3^3 + 0.1534x_3^2 - 0.0902x_3 + 0.0199) \quad \text{(with error of 7.77\%)} \tag{15}$$

$$y_{12} = -4.4200x_3 + 4.5739 \quad \text{(with error of 18.83\%)} \tag{16}$$

$$y_{13} = 10^{13}(0.0518x_3^8 - 0.3616x_3^7 + 1.1038x_3^6 - 1.9237x_3^5 + 2.0940x_3^4 - 1.4578x_3^3 + 0.6339x_3^2 - 0.1574x_3 + 0.0171 \quad \text{(with error of 2.40\%)} \tag{17}$$

$$y_{14} = -16.4255x_3^2 + 25.1580x_3 - 8.7004 \quad \text{(with error of 22.46\%)} \tag{18}$$

$$y_{15} = 10^{13}(0.0630x_3^8 - 0.4399x_3^7 + 1.3420x_3^6 - 2.3378x_3^5 + 2.5435x_3^4 - 1.7699x_3^3 + 0.7692x_3^2 - 0.1909x_3 + 0.0207 \quad \text{(with error of 3.93\%)} \tag{19}$$

$$y_{16} = 10^{13}(0.0292x_3^8 - 0.2041x_3^7 + 0.6238x_3^6 - 1.0886x_3^5 + 1.1866x_3^4 - 0.8272x_3^3 + 0.3602x_3^2 - 0.0896x_3 + 0.0097 \quad \text{(with error of 4.39\%)} \tag{20}$$

$$y_{17} = 10^{11}(-0.0209x_3^7 + 0.1286x_3^6 - 0.3396x_3^5 + 0.4978x_3^4 - 0.4373x_3^3 + 0.2304x_3^2 - 0.0674x_3 + 0.0084) \quad \text{(with error of 11.74\%)} \tag{21}$$

$$y_{18} = 10^{15}(-0.0047x_3^{10} + 0.0372x_3^9 - 0.1327x_3^8 + 0.2782x_3^7 - 0.3783x_3^6 + 0.3480x_3^5 - 0.2185x_3^4 + 0.0920x_3^3 - 0.0246x_3^2 + 0.0037x_3 - 0.0002) \quad \text{(with error of 3.80\%)} \tag{22}$$

$$y_{19} = 10^{15}(0.0073x_3^9 - 0.0564x_3^8 + 0.1941x_3^7 - 0.3890x_3^6 + 0.5008x_3^5 - 0.4296x_3^4 + 0.2455x_3^3 - 0.0901x_3^2 + 0.0193x_3 - 0.0018 \quad \text{(with error of 4.28\%)} \tag{23}$$

$$y_{20} = 10^{15}(0.0274x_3^9 - 0.2126x_3^8 + 0.7339x_3^7 - 1.4764x_3^6 + 1.9082x_3^5 - 1.6431x_3^4 + 0.9425x_3^3 - 0.3474x_3^2 + 0.0746x_3 - 0.0071 \quad \text{(with error of 2.92\%)} \tag{24}$$

$$y_{21} = -10^{13}(0.0127x_3^8 - 0.0896x_3^7 + 0.2759x_3^6 - 0.4853x_3^5 + 0.5331x_3^4 - 0.3746x_3^3 + 0.1644x_3^2 - 0.0412x_3 + 0.0045 \quad \text{(with error of 21.75\%)} \tag{25}$$

$$y_{22} = 10^{15}(0.0425x_3^9 - 0.3300x_3^8 + 1.1387x_3^7 - 2.2908x_3^6 + 2.9604x_3^5 - 2.5489x_3^4 + 1.4621x_3^3 - 0.5388x_3^2 + 0.1158x_3 - 0.0110 \quad \text{(with error of 13.52\%)} \tag{26}$$

$$y_{23} = -10^{13}(0.1656x_3^8 - 1.1553x_3^7 + 3.5247x_3^6 - 6.1401x_3^5 + 6.6803x_3^4 - 4.64836x_3^3 + 2.0201x_3^2 - 0.5013x_3 + 0.0544 \quad \text{(with error of 21.93\%)} \tag{27}$$

$$y_5 = 10^{13}(0.0029x_4^8 - 0.0201x_4^7 + 0.0612x_4^6 - 0.1068x_4^5 + 0.1162x_4^4 - 0.0809x_4^3 + 0.0352x_4^2 - 0.0087x_4 + 0.0009) \quad \text{(with error of 0.75\%)} \tag{28}$$

$$y_6 = 10^{15}(-0.0004x_4^{10} + 0.0029x_4^9 - 0.0092x_4^8 + 0.0160x_4^7 - 0.0163x_4^6 + 0.0087x_4^5 - 0.0003x_4^4 - 0.0028x_4^3 + 0.0018x_4^2 - 0.0005x_4 + 0.0001) \quad \text{(with error of 0.34\%)} \tag{29}$$

$$y_7 = 10^{13}(0.0005x_4^8 - 0.0033x_4^7 + 0.0100x_4^6 - 0.0174x_4^5 + 0.0190x_4^4 - 0.0132x_4^3 + 0.0058x_4^2 - 0.0014x_4 + 0.0002) \quad \text{(with error of 0.87\%)} \tag{30}$$

$$y_8 = 10^{15}(0.0167x_4^{10} - 0.1258x_4^9 + 0.4148x_4^8 - 0.7820x_4^7 + 0.9156x_4^6 - 0.6690x_4^5 + 0.2776x_4^4 - 0.0353x_4^3 - 0.0218x_4^2 + 0.0106x_4 - 0.0015) \quad \text{(with error of 1.21\%)} \tag{31}$$

$$y_9 = -10^{13}(0.0320x_4^8 - 0.2230x_4^7 + 0.6799x_4^6 - 1.1839x_4^5 + 1.2873x_4^4 - 0.8952x_4^3 + 0.3887x_4^2 - 0.0964x_4 + 0.0105) \quad \text{(with error of 9.90\%)} \tag{32}$$

$$y_{10} = -10^{13}(0.0549x_4^8 - 0.3833x_4^7 + 1.1690x_4^6 - 2.0360x_4^5 + 2.2144x_4^4 - 1.5402x_4^3 + 0.6690x_4^2 - 0.1659x_4 + 0.0180) \quad \text{(with error of 10.86\%)} \tag{33}$$

$$y_{11} = 10^9(0.0048x_4^6 - 0.0252x_4^5 + 0.0552x_4^4 - 0.0644x_4^3 + 0.0422x_4^2 + 0.0148x_4 + 0.0021) \quad \text{(with error of 3.57\%)} \tag{34}$$

$$y_{12} = -10^{13}(0.0757x_4^8 - 0.5282x_4^7 + 1.6112x_4^6 - 2.8066x_4^5 + 3.0529x_4^4 - 2.1238x_4^3 + 0.9227x_4^2 - 0.2289x_4 + 0.0248) \quad \text{(with error of 9.28\%)} \tag{35}$$

$$y_{13} = 10^{11}(-0.0487x_4^7 + 0.2997x_4^6 - 0.7894x_4^5 + 1.1540x_4^4 - 1.0112x_4^3 + 0.5311x_4^2 - 0.1548x_4 + 0.0193) \quad \text{(with error of 8.44\%)} \tag{36}$$

$$y_{14} = 10^{15}(0.0053x_4^9 - 0.0408x_4^8 + 0.1393x_4^7 - 0.2773x_4^6 + 0.3546x_4^5 - 0.3019x_4^4 + 0.1712x_4^3 - 0.0623x_4^2 + 0.0132x_4 - 0.0012) \quad \text{(with error of 3.88\%)} \tag{37}$$

$$y_{15} = 10^3(0.4151x_4^3 - 1.1333x_4^2 + 1.0240x_4 - 0.3056) \quad \text{(with error of 11.45\%)} \tag{38}$$

$$y_{16} = 10^{15}(-0.0088x_4^{10} + 0.0645x_4^9 - 0.2065x_4^8 + 0.3729x_4^7 - 0.4066x_4^6 + 0.2572x_4^5 - 0.0661x_4^4 - 0.0264x_4^3 + 0.0278x_4^2 - 0.0090x_4 + 0.0011) \quad \text{(with error of 6.11\%)} \tag{39}$$

$$y_{17} = 10^{13}(0.0245x_4^8 - 0.1708x_4^7 + 0.5203x_4^6 - 0.9051x_4^5 + 0.9833x_4^4 - 0.6831x_4^3 + 0.2964x_4^2 - 0.0734x_4 + 0.0080) \quad \text{(with error of 11.96\%)} \tag{40}$$

$$y_{18} = 10^{11}(-0.0144x_4^7 + 0.0885x_4^6 - 0.2334x_4^5 + 0.3416x_4^4 - 0.2997x_4^3 + 0.1576x_4^2 - 0.0460x_4 + 0.0058) \quad \text{(with error of 13.67\%)} \tag{41}$$

$$y_{19} = 10^{15}(0.0149x_4^9 - 0.1171x_4^8 + 0.4078x_4^7 - 0.8282x_4^6 + 1.0804x_4^5 - 0.9391x_4^4 + 0.5439x_4^3 - 0.2024x_4^2 + 0.0439x_4 - 0.0042) \quad \text{(with error of 3.87\%)} \tag{42}$$

$$y_{20} = 10^{15}(0.0060x_4^9 - 0.0469x_4^8 + 0.1629x_4^7 - 0.3302x_4^6 + 0.4301x_4^5 - 0.3731x_4^4 + 0.2157x_4^3 - 0.0801x_4^2 + 0.0173x_4 - 0.0017) \quad \text{(with error of 11.26\%)} \tag{43}$$

$$y_{21} = -10^{13}(0.1083x_4^8 - 0.7569x_4^7 + 2.3119x_4^6 - 4.0320x_4^5 + 4.3915x_4^4 - 3.0589x_4^3 + 1.3307x_4^2 - 0.3305x_4 + 0.0359) \quad \text{(with error of 15.56\%)} \tag{44}$$

$$y_{22} = -10^{13}(0.1053x_4^8 - 0.7356x_4^7 + 2.2472x_4^6 - 3.9198x_4^5 + 4.2699x_4^4 - 2.9746x_4^3 + 1.2942x_4^2 - 0.3215x_4 + 0.0349) \quad \text{(with error of 16.09\%)} \tag{45}$$

$$y_{23} = -10^{9}(0.0412x_4^6 - 0.2189x_4^5 + 0.4836x_4^4 - 0.5693x_4^3 + 0.3765x_4^2 + 0.1327x_4 + 0.0195) \quad \text{(with error of 25.29\%)} \tag{46}$$

$$y_{6} = 10^{15}(0.0006x_5^9 - 0.0041x_5^8 + 0.0015x_5^7 - 0.0169x_5^6 + 0.0127x_5^5 - 0.0020x_5^4 - 0.0046x_5^3 + 0.0041x_5^2 - 0.0015x_5 + 0.0002) \quad \text{(with error of 0.08\%)} \tag{47}$$

$$y_{7} = 10^{15}(0.0004x_5^9 - 0.0026x_5^8 + 0.0086x_5^7 - 0.0162x_5^6 + 0.0192x_5^5 - 0.0148x_5^4 + 0.0073x_5^3 - 0.0022x_5^2 + 0.0004x_5) \quad \text{(with error of 0.31\%)} \tag{48}$$

$$y_{8} = 10^{15}(0.0071x_5^9 - 0.0475x_5^8 + 0.1333x_5^7 - 0.1975x_5^6 + 0.1511x_5^5 - 0.0281x_5^4 - 0.0502x_5^3 + 0.0461x_5^2 + 0.0166x_5 + 0.0023) \quad \text{(with error of 1.38\%)} \tag{49}$$

$$y_{9} = 10^{12}(0.0561x_5^7 - 0.3624x_5^6 + 1.0022x_5^5 - 1.5392x_5^4 + 1.4179x_5^3 - 0.7835x_5^2 + 0.2404x_5 - 0.0316) \quad \text{(with error of 2.69\%)} \tag{50}$$

$$y_{10} = 10^{15}(0.0068x_5^9 - 0.0398x_5^8 + 0.0858x_5^7 - 0.05464x_5^6 - 0.1022x_5^5 + 0.2561x_5^4 - 0.2557x_5^3 + 0.1394x_5^2 - 0.0408x_5 + 0.0051) \quad \text{(with error of 5.91\%)} \tag{51}$$

$$y_{11} = 10^{5}(0.3563x_5^4 - 1.3260x_5^3 + 1.8491x_5^2 + 1.1451x_5 + 0.2657 \quad \text{(with error of 8.09\%)} \tag{52}$$

$$y_{12} = 10^{15}(0.0080x_5^9 - 0.0535x_5^8 + 0.1506x_5^7 - 0.2247x_5^6 + 0.1751x_5^5 - 0.0379x_5^4 - 0.0516x_5^3 + 0.0496x_5^2 - 0.0181x_5 + 0.0025) \quad \text{(with error of 13.20\%)} \tag{53}$$

$$y_{13} = 10^{12}(0.0038x_5^7 - 0.0237x_5^6 + 0.0642x_5^5 - 0.0963x_5^4 + 0.0867x_5^3 - 0.0468x_5^2 + 0.0140x_5 - 0.0018) \quad \text{(with error of 7.50\%)} \tag{54}$$

$$y_{14} = -35.3738x_5^2 - 59.2782x_5 - 23.8713 \quad \text{(with error of 22.59\%)} \tag{55}$$

$$y_{15} = 10^{12}(0.0346x_5^7 - 0.2231x_5^6 + 0.6161x_5^5 - 0.9447x_5^4 + 0.8689x_5^3 - 0.4794x_5^2 + 0.1469x_5 - 0.0193) \quad \text{(with error of 8.65\%)} \tag{56}$$

$$y_{16} = -10^{12}(0.0323x_5^7 - 0.2082x_5^6 + 0.5755x_5^5 - 0.8835x_5^4 + 0.8136x_5^3 - 0.4494x_5^2 + 0.1379x_5 - 0.0181) \quad \text{(with error of 6.34\%)} \tag{57}$$

$$y_{17} = -10^{12}(0.0430x_5^7 - 0.2779x_5^6 + 0.7687x_5^5 - 1.1808x_5^4 + 1.0880x_5^3 - 0.6013x_5^2 + 0.1846x_5 - 0.0243) \quad \text{(with error of 12.59\%)} \tag{58}$$

$$y_{18} = 10^{7}(-0.3842x_5^5 + 1.7913x_5^4 - 3.3391x_5^3 + 3.1106x_5^2 - 1.4481x_5 + 0.2695 \quad \text{(with error of 12.11\%)} \tag{59}$$

$$y_{19} = 10^{15}(0.0044x_5^9 - 0.0320x_5^8 + 0.1011x_5^7 - 0.1816x_5^6 + 0.2021x_5^5 - 0.1418x_5^4 + 0.0602x_5^3 - 0.0133x_5^2 + 0.0007x_5 + 0.0002) \quad \text{(with error of 2.65\%)} \tag{60}$$

$$y_{20} = 10^{3}(-1.2437x_5^3 + 3.4712x_5^2 - 3.2274x_5 + 1.0005 \quad \text{(with error of 19.14\%)} \tag{61}$$

$$y_{21} = 10^{15}(-0.0191x_5^8 + 0.1406x_5^7 - 0.4526x_5^6 + 0.8321x_5^5 - 0.9559x_5^4 + 0.7026x_5^3 - 0.3227x_5^2 + 0.0847x_5 - 0.0097) \quad \text{(with error of 12.57\%)} \tag{62}$$

$$y_{22} = 10^{15}(-0.0193x_5^8 + 0.1653x_5^7 - 0.5309x_5^6 + 0.9738x_5^5 - 1.1162x_5^4 + 0.8186x_5^3 - 0.3751x_5^2 + 0.0982x_5 - 0.0112) \quad \text{(with error of 12.18\%)} \tag{63}$$

$$y_{23} = 10^{15}(-0.0225x_5^8 + 0.1653x_5^7 - 0.5309x_5^6 + 0.9738x_5^5 - 1.1162x_5^4 + 0.8186x_5^3 - 0.3751x_5^2 + 0.0982x_5 - 0.0112) \quad \text{(with error of 13.13\%)} \tag{64}$$

$$y_7 = 10^{12}(0.0029x_6^7 - 0.0188x_6^6 + 0.0518x_6^5 - 0.0796x_6^4 + 0.0733x_6^3 - 0.0404x_6^2 + 0.0124x_6 - 0.0016) \quad \text{(with error of 0.73\%)} \tag{65}$$

$$y_8 = 10^{15}(0.0017x_6^8 - 0.0125x_6^7 + 0.0403x_6^6 - 0.0743x_6^5 + 0.0855x_6^4 - 0.0630x_6^3 + 0.0290x_6^2 - 0.0076x_6 + 0.0009) \quad \text{(with error of 1.45\%)} \tag{66}$$

$$y_9 = 10^{12}(0.0554x_6^7 - 0.3575x_6^6 + 0.9876x_6^5 - 1.5153x_6^4 + 1.3944x_6^3 - 0.7696x_6^2 + 0.2359x_6 - 0.0310) \quad \text{(with error of 8.53\%)} \tag{67}$$

$$y_{10} = -10^{12}(0.0839x_6^7 - 0.5407x_6^6 + 1.4937x_6^5 - 2.2916x_6^4 + 2.1086x_6^3 - 1.1636x_6^2 + 0.3566x_6 - 0.0468) \quad \text{(with error of 6.84\%)} \tag{68}$$

$$y_{11} = -10^{15}(0.0008x_6^9 - 0.0005x_6^8 - 0.0199x_6^7 + 0.0895x_6^6 - 0.1873x_6^5 + 0.2304x_6^4 - 0.1765x_6^3 + 0.0833x_6^2 - 0.0223x_6 + 0.0026) \quad \text{(with error of 6.29\%)} \tag{69}$$

$$y_{12} = -10^{12}(0.0822x_6^7 - 0.5300x_6^6 + 1.4642x_6^5 - 2.2464x_6^4 + 2.0672x_6^3 - 1.1409x_6^2 + 0.3497x_6 - 0.0459) \quad \text{(with error of 16.97\%)} \tag{70}$$

$$y_{13} = 10^5(-0.0584x_6^4 + 0.2191x_6^3 - 0.3080x_6^2 + 0.1925x_6 - 0.0451) \quad \text{(with error of 14.53\%)} \tag{71}$$

$$y_{14} = 10^8(0.0241x_6^5 - 0.1126x_6^4 + 0.2106x_6^3 - 0.1968x_6^2 + 0.0919x_6 - 0.0172) \quad \text{(with error of 13.02\%)} \tag{72}$$

$$y_{15} = -10^{12}(0.0496x_6^7 - 0.3199x_6^6 + 0.8838x_6^5 - 1.3562x_6^4 + 1.2482x_6^3 - 0.6890x_6^2 + 0.2112x_6 - 0.0277) \quad \text{(with error of 10.54\%)} \tag{73}$$

$$y_{16} = 10^{15}(0.0041x_6^8 - 0.0301x_6^7 + 0.0966x_6^6 - 0.1772x_6^5 + 0.2031x_6^4 - 0.1489x_6^3 + 0.0682x_6^2 - 0.0179x_6 + 0.0020) \quad \text{(with error of 5.52\%)} \tag{74}$$

$$y_{17} = 10^{15}(0.0098x_6^9 - 0.0453x_6^8 + 0.0356x_6^7 + 0.2028x_6^6 - 0.6620x_6^5 + 0.9574x_6^4 - 0.7988x_6^3 + 0.3975x_6^2 - 0.1102x_6 + 0.0132) \quad \text{(with error of 12.22\%)} \tag{75}$$

$$y_{18} = -10^{12}(0.0064x_6^7 - 0.0412x_6^6 + 0.1140x_6^5 - 0.1749x_6^4 + 0.1611x_6^3 - 0.0889x_6^2 + 0.0273x_6 - 0.0036) \quad \text{(with error of 12.78\%)} \tag{76}$$

$$y_{19} = 10^{10}(0.0475x_6^6 - 0.2638x_6^5 + 0.6098x_6^4 - 0.7515x_6^3 + 0.5207x_6^2 - 0.1923x_6 + 0.0296) \quad \text{(with error of 7.01\%)} \tag{77}$$

$$y_{20} = 10^{15}(0.0027x_6^9 - 0.0302x_6^8 + 0.1397x_6^7 - 0.3608x_6^6 + 0.5812x_6^5 - 0.6105x_6^4 + 0.4203x_6^3 - 0.1836x_6^2 + 0.0463x_6 - 0.0051) \quad \text{(with error of 3.32\%)} \tag{78}$$

$$y_{21} = 2.4420x_6 + 2.4079 \quad \text{(with error of 25.78\%)} \tag{79}$$

$$y_{22} = -2.1765x_6 + 2.1675 \quad \text{(with error of 24.75\%)} \tag{80}$$

$$y_{23} = 10^{15}(0.0303x_6^8 - 0.2223x_6^7 + 0.7128x_6^6 - 1.3059x_6^5 + 1.4948x_6^4 - 1.0947x_6^3 + 0.5009x_6^2 - 0.1309x_6 + 0.0150) \quad \text{(with error of 2.07\%)} \tag{81}$$

$$y_8 = 10^{15}(0.0278x_7^8 - 0.1869x_7^7 + 0.5340x_7^6 - 0.8350x_7^5 + 0.7613x_7^4 - 0.3899x_7^3 + 0.0887x_7^2 + 0.0041x_7 - 0.0041) \quad \text{(with error of 6.59)} \tag{82}$$

$$y_9 = -10^{15}(0.0475x_7^8 - 0.3243x_7^7 + 0.9443x_7^6 - 1.5186x_7^5 + 1.4493x_7^4 - 0.8109x_7^3 + 0.2363x_7^2 - 0.0207x_7 - 0.0031) \quad \text{(with error of 13.61)} \tag{83}$$

$$y_{10} = 10^{13}(-0.0347x_7^6 + 0.2045x_7^5 - 0.5030x_7^4 + 0.6597x_7^3 - 0.4866x_7^2 + 0.1915x_7 - 0.0314) \quad \text{(with error of 45.14\%)} \tag{84}$$

$$y_{11} = -10^{15}(0.0300x_7^8 - 0.1963x_7^7 + 0.5384x_7^6 - 0.7888x_7^5 + 0.6369x_7^4 - 0.2390x_7^3 - 0.0116x_7^2 + 0.0396x_7 - 0.0093) \quad \text{(with error of 16.46)} \tag{85}$$

$$y_{12} = -10^{15}(0.0753x_7^8 - 0.5011x_7^7 + 1.4085x_7^6 - 2.1465x_7^5 + 1.8699x_7^4 - 0.8647x_7^3 + 0.1263x_7^2 + 0.0487x_7 - 0.0164) \quad \text{(with error of 23.03)} \tag{86}$$

$$y_{13} = 10^{13}(-0.0350x_7^6 + 0.2064x_7^5 - 0.5078x_7^4 + 0.6663x_7^3 - 0.4918x_7^2 + 0.1936x_7 - 0.0317) \quad \text{(with error of 21.35\%)} \tag{87}$$

$$y_{14} = -10^{15}(-0.0333x_7^8 + 0.2190x_7^7 - 0.6037x_7^6 + 0.8903x_7^5 - 0.7268x_7^4 + 0.2808x_7^3 + 0.0060x_7^2 - 0.0424x_7 + 0.0102) \quad \text{(with error of 30.28\%)} \tag{88}$$

$$y_{15} = -10^{13}(-0.0413x_7^6 + 0.2442x_7^5 - 0.6011x_7^4 + 0.7891x_7^3 - 0.5828x_7^2 + 0.2295x_7 - 0.0377) \quad \text{(with error of 15.42\%)} \tag{89}$$

$$y_{16} = 10^{8}(0.0541x_7^4 - 0.2135x_7^3 + 0.3162x_7^2 - 0.2080x_7 + 0.0513) \quad \text{(with error of 22.68\%)} \tag{90}$$

$$y_{17} = 10^{8}(-0.0270x_7^4 + 0.1053x_7^3 - 0.1543x_7^2 + 0.1004x_7 - 0.0245) \quad \text{(with error of 9.58\%)} \tag{91}$$

$$y_{18} = -10^{13}(-0.0291x_7^6 + 0.1722x_7^5 - 0.4242x_7^4 + 0.5573x_7^3 - 0.4118x_7^2 + 0.1623x_7 - 0.0267) \quad \text{(with error of 15.13\%)} \tag{92}$$

$$y_{19} = -10^{12}(0.0243x_6^7 - 0.1573x_6^6 + 0.4359x_6^5 - 0.6707x_6^4 + 0.6189x_6^3 - 0.3425x_6^2 + 0.1053x_6 - 0.0139) \quad \text{(with error of 7.09\%)} \tag{93}$$

$$y_{20} = 10^{15}(-0.0158x_7^8 + 0.0999x_7^7 - 0.2604x_7^6 + 0.3477x_7^5 - 0.2248x_7^4 + 0.0172x_7^3 + 0.0702x_7^2 - 0.0419x_7 + 0.0079 \quad \text{(with error of 4.47\%)} \tag{94}$$

$$y_{21} = 10^{3}(-0.7980x_7^2 + 1.5607x_7 - 0.7627) \quad \text{(with error of 27.74\%)} \tag{95}$$

$$y_{22} = 10^{3}(-0.6919x_7^2 + 1.3519x_7 - 0.6601) \quad \text{(with error of 26.65\%)} \tag{96}$$

$$y_{23} = 10^{14}(0.0551x_7^7 - 0.3837x_7^6 + 1.1445x_7^5 - 1.8964x_7^4 + 1.8853x_7^3 - 1.1245x_7^2 + 0.3726x_7 - 0.0529) \quad \text{(with error of 1.25\%)} \tag{97}$$

$$y_9 = 10^{13}(0.0544x_8^7 - 0.3530x_8^6 + 0.9785x_8^5 - 1.5013x_8^4 + 1.3759x_8^3 - 0.7528x_8^2 + 0.2275x_8 - 0.0293) \quad \text{(with error of 15.16\%)} \tag{98}$$

$$y_{10} = 10^{13}(0.0290x_8^7 - 0.1880x_8^6 + 0.5204x_8^5 - 0.7973x_8^4 + 0.7297x_8^3 - 0.3987x_8^2 + 0.1203x_8 - 0.0155) \quad \text{(with error of 8.74\%)} \tag{99}$$

$$y_{11} = 10^{10}(0.1902x_8^6 - 1.0453x_8^5 + 2.3820x_8^4 - 2.8793x_8^3 + 1.9454x_8^2 - 0.6958x_8 + 0.1028) \quad \text{(with error of 9.74\%)} \tag{100}$$

$$y_{12} = 10^{13}(0.0638x_8^7 - 0.4140x_8^6 + 1.1472x_8^5 - 1.7593x_8^4 + 1.6117x_8^3 - 0.8814x_8^2 + 0.2663x_8 - 0.0342) \quad \text{(with error of 3.68\%)} \tag{101}$$

$$y_{13} = 10^{10}(0.3883x_8^6 - 2.1374x_8^5 + 4.8792x_8^4 - 5.9078x_8^3 + 3.9983x_8^2 - 1.4324x_8 + 0.2119) \quad \text{(with error of 15.42\%)} \tag{102}$$

$$y_{14} = 10^8(0.3920x_8^5 - 1.7743x_8^4 + 3.1896x_8^3 - 2.8429x_8^2 + 1.2542x_8 - 0.2186) \quad \text{(with error of 19.47\%)} \tag{103}$$

$$y_{15} = 10^{10}(0.1835x_8^6 - 1.0083x_8^5 + 2.2972x_8^4 - 2.7761x_8^3 + 1.8753x_8^2 - 0.6706x_8 + 0.0990) \quad \text{(with error of 6.40\%)} \tag{104}$$

$$y_{16} = 10^{15}(-0.0002x_8^8 + 0.0011x_8^7 - 0.0018x_8^6 + 0.0001x_8^5 + 0.0038x_8^4 - 0.0056x_8^3 + 0.0038x_8^2 - 0.0013x_8 + 0.0002) \quad \text{(with error of 11.99\%)} \tag{105}$$

$$y_{17} = 10^{15}(-0.0017x_8^8 + 0.012x_8^7 - 0.0368x_8^6 + 0.0638x_8^5 - 0.06878x_8^4 - 0.0470x_8^3 - 0.0199x_8^2 + 0.0048x_8 - 0.0005) \quad \text{(with error of 5.99\%)} \tag{106}$$

$$y_{18} = -10^8(0.1673x_8^5 - 0.7569x_8^4 + 1.3606x_8^3 - 1.2127x_8^2 + 0.5350x_8 - 0.0932) \quad \text{(with error of 13.51\%)} \tag{107}$$

$$y_{19} = 0.7973x_8 - 0.1898 \quad \text{(with error of 14.88\%)} \tag{108}$$

$$y_{20} = -9.1630x_8^2 + 14.0679x_8 - 4.1517 \quad \text{(with error of 18.57\%)} \tag{109}$$

$$y_{21} = 10^{13}(0.2521x_8^7 - 1.6372x_8^6 + 4.5410x_8^5 - 6.9704x_8^4 + 6.3915x_8^3 - 3.4987x_8^2 + 1.0578x_8 - 0.1361) \quad \text{(with error of 13.22\%)} \tag{110}$$

$$y_{22} = 10^{15}(-0.0013x_8^8 + 0.0125x_8^7 - 0.0491x_8^6 + 0.1068x_8^5 - 0.1413x_8^4 - 0.1172x_8^3 - 0.0596x_8^2 + 0.0171x_8 - 0.0021) \quad \text{(with error of 9.47\%)} \tag{111}$$

$$y_{23} = 10^{15}(-0.0236x_8^8 + 0.1768x_8^7 - 0.5789x_8^6 + 1.0802x_8^5 - 1.2555x_8^4 - 0.9305x_8^3 - 0.4293x_8^2 + 0.1126x_8 - 0.0129) \quad \text{(with error of 10.24\%)} \tag{112}$$

$$y_{10} = 10^7(0.0230x_9^{10} - 0.1572x_9^9 + 0.4755x_9^8 - 0.8345x_9^7 + 0.9371x_9^6 - 0.7001x_9^5 + 0.3497x_9^4 - 0.1140x_9^3 + 0.0228x_9^2 - 0.0024x_9 + 0.0001) \quad \text{(with error of 1.87\%)} \tag{113}$$

$$y_{11} = 10^{10}(0.0315x_9^7 - 0.1476x_9^6 + 0.2832x_9^5 - 0.2850x_9^4 + 0.1599x_9^3 - 0.0486x_9^2 + 0.0071x_9 - 0.0003) \quad \text{(with error of 5.05\%)} \tag{114}$$

$$y_{12} = 10^{10}(0.0519x_9^{10} - 0.3551x_9^9 + 1.0733x_9^8 - 1.8819x_9^7 + 2.1119x_9^6 - 2.5767x_9^5 + 0.7870x_9^4 - 0.2564x_9^3 + 0.0512x_9^2 - 0.0055x_9 + 0.0002) \quad \text{(with error of 2.42\%)} \tag{115}$$

$$y_{13} = 10^{10}(-0.0017x_9^{10} + 0.0116x_9^9 - 0.0354x_9^8 + 0.0627x_9^7 - 0.0711x_9^6 + 0.0536x_9^5 - 0.0270x_9^4 + 0.0089x_9^3 - 0.0018x_9^2 + 0.0002x_9) \quad \text{(with error of 0.23\%)} \tag{116}$$

$$y_{14} = 10^{10}(-0.0388x_9^{10} + 0.2659x_9^9 - 0.8043x_9^8 + 1.4116x_9^7 - 1.5855x_9^6 + 1.1847x_9^5 - 0.5919x_9^4 + 0.1930x_9^3 - 0.0386x_9^2 + 0.0041x_9 - 0.0002) \quad \text{(with error of 15.84\%)} \tag{117}$$

$$y_{15} = 10^{10}(-0.0171x_9^{10} + 0.1170x_9^9 - 0.3534x_9^8 + 0.6194x_9^7 - 0.6949x_9^6 + 0.5186x_9^5 - 0.2588x_9^4 + 0.0843x_9^3 - 0.0168x_9^2 + 0.0018x_9 - 0.0001) \quad \text{(with error of 2.44\%)} \tag{118}$$

$$y_{16} = 10^{10}(-0.0034x_9^{10} + 0.0232x_9^9 - 0.0693x_9^8 + 0.1200x_9^7 - 0.1330x_9^6 + 0.0981x_9^5 - 0.0484x_9^4 + 0.0156x_9^3 - 0.0031x_9^2 + 0.0003x_9) \quad \text{(with error of 7.44\%)} \tag{119}$$

$$y_{17} = 10^{10}(-0.0433x_9^{10} + 0.2955x_9^9 - 0.8915x_9^8 + 1.5603x_9^7 - 1.7479x_9^6 + 1.3027x_9^5 - 0.6492x_9^4 + 0.2112x_9^3 - 0.0421x_9^2 + 0.0045x_9 - 0.0002) \quad \text{(with error of 6.34\%)} \tag{120}$$

$$y_{18} = 10^{10}(-0.0029x_9^{10} + 0.0198x_9^9 - 0.0590x_9^8 + 0.1022x_9^7 - 0.1132x_9^6 + 0.0834x_9^5 - 0.0411x_9^4 + 0.0132x_9^3 - 0.0026x_9^2 + 0.0003x_9) \quad \text{(with error of 8.15\%)} \tag{121}$$

$$y_{19} = 10^8(0.0251x_9^9 - 0.1519x_9^8 + 0.3990x_9^7 - 0.5947x_9^6 + 0.5513x_9^5 - 0.3269x_9^4 + 0.1224x_9^3 - 0.0273x_9^2 + 0.0032x_9 - 0.0001) \quad \text{(with error of 4.21\%)} \tag{122}$$

$$y_{20} = 10^8(-0.0284x_9^9 + 0.1717x_9^8 - 0.4506x_9^7 + 0.6710x_9^6 - 0.6215x_9^5 + 0.3682x_9^4 - 0.1377x_9^3 + 0.0307x_9^2 - 0.0036x_9 + 0.0002) \quad \text{(with error of 16.73\%)} \tag{123}$$

$$y_{21} = 10^{10}(-0.0296x_9^{10} + 0.2031x_9^9 - 0.6153x_9^8 + 1.0816x_9^7 - 1.2168x_9^6 + 0.9106x_9^5 - 0.4556x_9^4 + 0.1488x_9^3 - 0.0298x_9^2 + 0.0032x_9 - 0.0001) \quad \text{(with error of 24.94\%)} \tag{124}$$

$$y_{22} = 10^{10}(-0.0333x_9^{10} + 0.2284x_9^9 - 0.6916x_9^8 + 1.2152x_9^7 - 1.3665x_9^6 + 1.0222x_9^5 - 0.5112x_9^4 + 0.1669x_9^3 - 0.0334x_9^2 + 0.0036x_9 - 0.0001) \quad \text{(with error of 20.73\%)} \tag{125}$$

$$y_{23} = 10^5(0.1447x_9^7 - 0.6583x_9^6 + 1.2356x_9^5 - 1.2269x_9^4 + 0.6840x_9^3 - 0.2080x_9^2 + 0.0303x_9 - 0.0015) \quad \text{(with error of 26.32\%)} \tag{126}$$

$$y_{11} = 10^9(0.0243x_{10}^8 - 0.1515x_{10}^7 + 0.4067x_{10}^6 - 0.6130x_{10}^5 + 0.5649x_{10}^4 - 0.3238x_{10}^3 + 0.1116x_{10}^2 - 0.0207x_{10} + 0.0015) \quad \text{(with error of 6.76\%)} \tag{127}$$

$$y_{12} = 10^{11}(0.0263x_{10}^9 - 0.1879x_{10}^8 + 0.5900x_{10}^7 - 1.0664x_{10}^6 + 1.2193x_{10}^5 - 0.9110x_{10}^4 + 0.4421x_{10}^3 - 0.1332x_{10}^2 + 0.0222x_{10} - 0.0015) \quad \text{(with error of 0.82\%)} \tag{128}$$

$$y_{13} = 10^{11}(-0.0708x_{10}^9 + 0.5065x_{10}^8 - 1.5922x_{10}^7 + 2.8807x_{10}^6 - 3.2969x_{10}^5 + 2.4655x_{10}^4 - 1.1977x_{10}^3 + 0.3611x_{10}^2 - 0.0603x_{10} + 0.0041) \quad \text{(with error of 10.43\%)} \tag{129}$$

$$y_{14} = 10^{11}(-0.0434x_{10}^9 + 0.3108x_{10}^8 - 0.9772x_{10}^7 + 1.7686x_{10}^6 - 2.0249x_{10}^5 + 1.5148x_{10}^4 - 0.7361x_{10}^3 + 0.2220x_{10}^2 - 0.0371x_{10} + 0.0025) \quad \text{(with error of 14.09\%)} \tag{130}$$

$$y_{15} = 10^{11}(-0.0161x_{10}^9 + 0.1149x_{10}^8 - 0.3613x_{10}^7 + 0.6540x_{10}^6 - 0.7488x_{10}^5 + 0.5601x_{10}^4 - 0.2722x_{10}^3 + 0.0821x_{10}^2 - 0.0137x_{10} + 0.0009) \quad \text{(with error of 4.53\%)} \tag{131}$$

$$y_{16} = 10^{17}(0.0042x_{10}^{10} - 0.0339x_{10}^9 + 0.1221x_{10}^8 - 0.2582x_{10}^7 + 0.3540x_{10}^6 - 0.3278x_{10}^5 + 0.2069x_{10}^4 - 0.0875x_{10}^3 + 0.0235x_{10}^2 - 0.0036x_{10} + 0.0002) \quad \text{(with error of 60.38\%)} \tag{132}$$

$$y_{17} = 10^{11}(-0.0108x_{10}^9 + 0.0770x_{10}^8 - 0.2410x_{10}^7 + 0.4343x_{10}^6 - 0.4952x_{10}^5 + 0.3689x_{10}^4 - 0.1786x_{10}^3 + 0.0537x_{10}^2 - 0.0089x_{10} + 0.0006) \quad \text{(with error of 8.88\%)} \tag{133}$$

$$y_{18} = 10^7(0.0071x_{10}^7 - 0.0395x_{10}^6 + 0.0917x_{10}^5 - 0.1151x_{10}^4 + 0.0839x_{10}^3 - 0.0350x_{10}^2 + 0.0076x_{10} - 0.0006) \quad \text{(with error of 11.48\%)} \tag{134}$$

$$y_{19} = 1.3676x_{10}^2 - 1.8268x_{10} + 1.0608 \quad \text{(with error of 22.07\%)} \tag{135}$$

$$y_{20} = -10^7(0.0175x_{10}^7 - 0.0939x_{10}^6 + 0.2113x_{10}^5 - 0.2578x_{10}^4 + 0.1828x_{10}^3 - 0.0744x_{10}^2 + 0.0157x_{10} - 0.0013) \quad \text{(with error of 16.67\%)} \tag{136}$$

$$y_{21} = 5.1263x_{10}^3 - 0.95237x_{10}^2 + 5.4239x_{10} - 0.7038 \quad \text{(with error of 22.89\%)} \tag{137}$$

$$y_{22} = 99.3243x_{10}^4 - 269.4538x_{10}^3 + 256.4748x_{10}^2 - 96.8644x_{10} + 10.8853 \quad \text{(with error of 20.29\%)} \tag{138}$$

$$y_{23} = 10^7(0.0383x_{10}^7 - 0.1983x_{10}^6 + 0.4317x_{10}^5 - 0.5092x_{10}^4 + 0.3492x_{10}^3 - 0.1376x_{10}^2 + 0.0282x_{10} - 0.0022) \quad \text{(with error of 8.58\%)} \tag{139}$$

$$y_{12} = 10^{11}(0.0154x_{11}^{10} - 0.1081x_{11}^9 + 0.3357x_{11}^8 - 0.6034x_{11}^7 + 0.6913x_{11}^6 - 0.5236x_{11}^5 + 0.2625x_{11}^4 - 0.0845x_{11}^3 + 0.0163x_{11}^2) \quad \text{(with error of 3.86\%)} \tag{140}$$

$$y_{13} = 10^3(0.0858x_{11}^4 - 0.2356x_{11}^4 + 0.2342x_{11}^3 - 0.10031x_{11}^2 + 0.0177x_{11} - 0.0007) \quad \text{(with error of 10.27\%)} \tag{141}$$

$$y_{14} = 10^{11}(-0.0140x_{11}^{10} + 0.0984x_{11}^{9} - 0.3058x_{11}^{8} + 0.5500x_{11}^{7} - 0.6307x_{11}^{6} + 0.4781x_{11}^{5} - 0.2399x_{11}^{4} + 0.0773x_{11}^{3} - 0.0149x_{11}^{2} + 0.0015x_{11}) \quad \text{(with error of 14.17\%)} \tag{142}$$

$$y_{15} = 10^{11}(0.0108x_{11}^{10} - 0.0757x_{11}^{9} + 0.2335x_{11}^{8} - 0.4170x_{11}^{7} + 0.4747x_{11}^{6} - 0.3571x_{11}^{5} + 0.1779x_{11}^{4} - 0.0569x_{11}^{3} + 0.0109x_{11}^{2} - 0.0011x_{11} - 0.0001) \quad \text{(with error of 9.54\%)} \tag{143}$$

$$y_{16} = 10^{7}(-0.0596x_{11}^{8} + 0.3193x_{11}^{7} - 0.7239x_{11}^{6} + 0.8994x_{11}^{5} - 0.6607x_{11}^{4} + 0.2876x_{11}^{3} - 0.0698x_{11}^{2} + 0.0081x_{11}^{2} - 0.0003) \quad \text{(with error of 5.38\%)} \tag{144}$$

$$y_{17} = 10^{9}(0.0326x_{11}^{9} - 0.2036x_{11}^{8} + 0.5515x_{11}^{7} - 0.8448x_{11}^{6} + 0.7999x_{11}^{5} - 0.4795x_{11}^{4} + 0.1785x_{11}^{3} - 0.0385x_{11}^{2} + 0.0041x_{11}^{2} - 0.0002) \quad \text{(with error of 8.12\%)} \tag{145}$$

$$y_{18} = 10^{11}(0.0078x_{11}^{10} - 0.0539x_{11}^{9} + 0.1655x_{11}^{8} - 0.2939x_{11}^{7} + 0.3327x_{11}^{6} - 0.2490x_{11}^{5} + 0.1234x_{11}^{4} - 0.0393x_{11}^{3} + 0.0075x_{11}^{2} - 0.0007x_{11}) \quad \text{(with error of 6.57\%)} \tag{146}$$

$$y_{19} = 10^{11}(0.0035x_{11}^{10} - 0.0250x_{11}^{9} + 0.0779x_{11}^{8} - 0.1406x_{11}^{7} + 0.1617x_{11}^{6} - 0.1229x_{11}^{5} + 0.0619x_{11}^{4} - 0.0200x_{11}^{3} + 0.0039x_{11}^{2} - 0.0004x_{11}) \quad \text{(with error of 0.95\%)} \tag{147}$$

$$y_{20} = 10^{11}(0.0004x_{11}^{10} - 0.0031x_{11}^{9} + 0.0107x_{11}^{8} - 0.0209x_{11}^{7} + 0.0258x_{11}^{6} - 0.0209x_{11}^{5} + 0.0111x_{11}^{4} - 0.0038x_{11}^{3} + 0.0008x_{11}^{2} - 0.0001x_{11}) \quad \text{(with error of 5.29\%)} \tag{148}$$

$$y_{21} = 10^{11}(0.0291x_{11}^{10} - 0.2039x_{11}^{9} + 0.6291x_{11}^{8} - 1.1236x_{11}^{7} + 1.2792x_{11}^{6} - 0.9627x_{11}^{5} + 0.4797x_{11}^{4} - 0.1536x_{11}^{3} + 0.0294x_{11}^{2} - 0.0029x_{11} + 0.0001) \quad \text{(with error of 36.41\%)} \tag{149}$$

$$y_{22} = -9.2682x_{11}^{4} + 18.0352x_{11}^{3} - 11.1493x_{11}^{2} + 2.5850x_{11} - 0.1184 \quad \text{(with error of 21.74\%)} \tag{150}$$

$$y_{23} = 10^{11}(-0.0329x_{11}^{10} + 0.2297x_{11}^{9} - 0.7078x_{11}^{8} + 1.2625x_{11}^{7} - 1.4355x_{11}^{6} + 1.0790x_{11}^{5} - 0.5369x_{11}^{4} + 0.1717x_{11}^{3} - 0.0328x_{11}^{2} + 0.0032x_{11} - 0.0001) \quad \text{(with error of 21.83\%)} \tag{151}$$

$$y_{13} = -10^{6}(0.1338x_{12}^{7} - 0.6826x_{12}^{6} + 1.4616x_{12}^{5} - 1.6956x_{12}^{4} + 1.1437x_{12}^{3} - 0.4439x_{12}^{2} + 0.0901x_{12} - 0.0071) \quad \text{(with error of 10.43\%)} \tag{152}$$

$$y_{14} = 10^{17}(0.0377x_{12}^{10} - 0.2963x_{12}^{9} + 1.0389x_{12}^{8} - 2.1348x_{12}^{7} + 2.8419x_{12}^{6} - 2.5549x_{12}^{5} + 1.5658x_{12}^{4} - 0.6430x_{12}^{3} + 0.1681x_{12}^{2} - 0.0250x_{12} + 0.0016) \quad \text{(with error of 7.41\%)} \tag{153}$$

$$y_{15} = 10^{7}(-0.1274x_{12}^{8} + 0.7601x_{12}^{7} - 1.9542x_{12}^{6} + 2.8201x_{12}^{5} - 2.4890x_{12}^{4} + 1.3680x_{12}^{3} - 0.4532x_{12}^{2} + 0.0815x_{12} - 0.0059) \quad \text{(with error of 4.07\%)} \tag{154}$$

$$y_{16} = 10^{6}(0.0158x_{12}^{7} - 0.0796x_{12}^{6} + 0.1680x_{12}^{5} - 0.1920x_{12}^{4} + 0.1275x_{12}^{3} - 0.0487x_{12}^{2} + 0.0097x_{12} - 0.0008) \quad \text{(with error of 11.86\%)} \tag{155}$$

$$y_{17} = 10^{6}(0.0706x_{12}^{7} - 0.3612x_{12}^{6} + 0.7763x_{12}^{5} - 0.9054x_{12}^{4} + 0.6148x_{12}^{3} - 0.2405x_{12}^{2} + 0.0492x_{12} - 0.0039) \quad \text{(with error of 8.88\%)} \tag{156}$$

$$y_{18} = -10^{6}(0.1172x_{12}^{7} - 0.6081x_{12}^{6} + 1.3247x_{12}^{5} - 1.5640x_{12}^{4} + 1.0738x_{12}^{3} - 0.4241x_{12}^{2} + 0.0875x_{12} - 0.0070) \quad \text{(with error of 11.48\%)} \tag{157}$$

$$y_{19} = 1.6289x_{12}^{2} - 2.1370x_{12} + 1.1277 \quad \text{(with error of 22.66\%)} \tag{158}$$

$$y_{20} = 57.8441x_{12}^{4} - 158.8260x_{12}^{3} + 151.9245x_{12}^{2} - 57.4083x_{12} + 7.2865 \quad \text{(with error of 16.66\%)} \tag{159}$$

$$y_{21} = 0.2918x_{12} - 0.0279 \quad \text{(with error of 25.06\%)} \tag{160}$$

$$y_{22} = 0.2494x_{12} + 0.0045 \quad \text{(with error of 23.28\%)} \tag{161}$$

$$y_{23} = 10^{10}(0.0388x_{12}^9 - 0.2679x_{12}^8 + 0.8121x_{12}^7 - 1.4163x_{12}^6 + 1.5619x_{12}^5 - 1.1256x_{12}^4 + 0.5274x_{12}^3 - 0.1537x_{12}^2 + 0.0250x_{12} - 0.0017) \quad \text{(with error of 12.90\%)} \tag{162}$$

$$y_{14} = 10^{11}(-0.0399x_{13}^{10} + 0.2525x_{13}^9 - 0.7065x_{13}^8 + 1.1482x_{13}^7 - 1.1967x_{13}^6 + 0.8319x_{13}^5 - 0.3880x_{13}^4 + 0.1188x_{13}^3 - 0.0225x_{13}^2 + 0.0023x_{13} - 0.0001) \quad \text{(with error of 3.52\%)} \tag{163}$$

$$y_{15} = 10^7(0.0206x_{13}^8 - 0.0958x_{13}^7 + 0.1882x_{13}^6 - 0.2033x_{13}^5 + 0.1313x_{13}^4 - 0.0513x_{13}^3 + 0.0117x_{13}^2 - 0.0014x_{13} + 0.0001 \quad \text{(with error of 3.52\%)} \tag{164}$$

$$y_{16} = 10^{10}(0.0118x_{13}^9 - 0.0661x_{13}^8 + 0.1616x_{13}^7 - 0.2246x_{13}^6 + 0.1946x_{13}^5 - 0.1082x_{13}^4 + 0.0382x_{13}^3 - 0.0081x_{13}^2 + 0.0009x_{13}) \quad \text{(with error of 6.66\%)} \tag{165}$$

$$y_{17} = 10^7(0.1171x_{13}^8 - 0.5649x_{13}^7 + 1.1575x_{13}^6 - 1.3096x_{13}^5 + 0.8880x_{13}^4 - 0.3652x_{13}^3 + 0.0872x_{13}^2 - 0.0107x_{13} + 0.0005 \quad \text{(with error of 10.54\%)} \tag{166}$$

$$y_{18} = 10^{11}(-0.0053x_{13}^{10} + 0.0310x_{13}^9 - 0.0802x_{13}^8 + 0.1195x_{13}^7 - 0.1132x_{13}^6 + 0.0710x_{13}^5 - 0.0296x_{13}^4 + 0.0080x_{13}^3 - 0.0013x_{13}^2 + 0.0001x_{13}) \quad \text{(with error of 0.28\%)} \tag{167}$$

$$y_{19} = 10^{11}(0.0068x_{13}^{10} - 0.0447x_{13}^9 + 0.1294x_{13}^8 - 0.2175x_{13}^7 + 0.2342x_{13}^6 - 0.1679x_{13}^5 + 0.0806x_{13}^4 - 0.0254x_{13}^3 + 0.0049x_{13}^2 - 0.0005x_{13}) \quad \text{(with error of 5.47\%)} \tag{168}$$

$$y_{20} = 10^{11}(0.0067x_{13}^{10} - 0.0400x_{13}^9 + 0.1059x_{13}^8 - 0.1624x_{13}^7 + 0.1591x_{13}^6 - 0.1036x_{13}^5 + 0.0451x_{13}^4 - 0.0129x_{13}^3 + 0.0023x_{13}^2 - 0.0002x_{13}) \quad \text{(with error of 12.50\%)} \tag{169}$$

$$y_{21} = 10^{11}(0.1523x_{13}^{10} - 0.9639x_{13}^9 + 2.6974x_{13}^8 - 4.3858x_{13}^7 + 4.5728x_{13}^6 - 3.1801x_{13}^5 + 1.4840x_{13}^4 - 0.4543x_{13}^3 + 0.0859x_{13}^2 - 0.0088x_{13} + 0.0003) \quad \text{(with error of 1.79\%)} \tag{170}$$

$$y_{22} = 10^{11}(0.1422x_{13}^{10} - 0.9001x_{13}^9 + 2.5209x_{13}^8 - 4.1016x_{13}^7 + 4.2796x_{13}^6 - 2.9782x_{13}^5 + 1.3908x_{13}^4 - 0.4260x_{13}^3 + 0.0806x_{13}^2 - 0.0083x_{13} + 0.0003) \quad \text{(with error of 2.33\%)} \tag{171}$$

$$y_{23} = 10^{11}(0.0173x_{13}^{10} - 0.1005x_{13}^9 + 0.2552x_{13}^8 - 0.3723x_{13}^7 + 0.3438x_{13}^6 - 0.2087x_{13}^5 + 0.0836x_{13}^4 - 0.0216x_{13}^3 + 0.0034x_{13}^2 - 0.0003x_{13}) \quad \text{(with error of 1.61\%)} \tag{172}$$

$$y_{15} = 10^9(0.0225x_{14}^9 - 0.1343x_{14}^8 + 0.3468x_{14}^7 - 0.5074x_4^6 + 0.4605x_{14}^5 - 0.2666x_{14}^4 + 0.0972x_{14}^3 - 0.0211x_{14}^2 + 0.0024x_{14} - 0.0001 \quad \text{(with error of 0.65\%)} \tag{173}$$

$$y_{16} = 10^{15}(-0.0012x_{14}^{10} + 0.0084x_{14}^9 - 0.0255x_{14}^8 + 0.0445x_{14}^7 - 0.0496x_{14}^6 + 0.0368x_{14}^5 - 0.0182x_{14}^4 + 0.0058x_{14}^3 - 0.0011x_{14}^2 + 0.0001x_{14}) \quad \text{(with error of 0.17\%)} \tag{174}$$

$$y_{17} = 10^9(0.0151x_{14}^9 - 0.0896x_{14}^8 + 0.2307x_{14}^7 - 0.3364x_{14}^6 + 0.3044x_{14}^5 - 0.1757x_{14}^4 + 0.0639x_{14}^3 - 0.0139x_{14}^2 + 0.0016x_{14} - 0.0001 \quad \text{(with error of 11.74\%)} \tag{175}$$

$$y_{18} = 10^9(0.0301x_{14}^9 - 0.1795x_{14}^8 + 0.4643x_{14}^7 - 0.6802x_{14}^6 + 0.6181x_{14}^5 - 0.3582x_{14}^4 + 0.1308x_{14}^3 - 0.0285x_{14}^2 + 0.0033x_{14} - 0.0001 \quad \text{(with error of 9.57\%)} \tag{176}$$

$$y_{19} = 10^{15}(-0.0051x_{14}^{10} + 0.0347x_{14}^9 - 0.1046x_{14}^8 + 0.1828x_{14}^7 - 0.2040x_{14}^6 + 0.1511x_{14}^5 - 0.0746x_{14}^4 + 0.0240x_{14}^3 - 0.0047x_{14}^2 + 0.0005x_{14}) \quad \text{(with error of 1.81\%)} \tag{177}$$

$$y_{20} = 10^4(0.0839x_{14}^6 - 0.3087x_{14}^5 + 0.4433x_{14}^4 - 0.3110x_{14}^3 + 0.1084x_{14}^2 - 0.0167x_{14} + 0.0009) \quad \text{(with error of 17.06\%)} \tag{178}$$

$$y_{21} = 10^9(0.0037x_{14}^9 - 0.0233x_{14}^8 + 0.0631x_{14}^7 - 0.0965x_4^6 + 0.0915x_{14}^5 - 0.0553x_{14}^4 + 0.0210x_{14}^3 - 0.0047x_{14}^2 + 0.0006x_{14} \quad \text{(with error of 0.41\%)} \tag{179}$$

$$y_{22} = 10^9(0.0022x_{14}^9 - 0.0139x_{14}^8 + 0.0386x_{14}^7 - 0.0606x_4^6 + 0.0588x_{14}^5 - 0.0363x_{14}^4 + 0.0140x_{14}^3 - 0.0032x_{14}^2 + 0.0004x_{14} \quad \text{(with error of 0.37\%)} \tag{180}$$

$$y_{23} = -1.5105x_{14}^2 + 1.9019x_{14} - 0.0867 \quad \text{(with error of 31.41\%)} \tag{181}$$

$$y_{16} = 10^{10}(0.0550x_{15}^{10} - 0.3271x_{15}^9 + 0.8642x_{15}^8 - 1.3364x_{15}^7 + 1.3401x_{15}^6 - 0.9110x_{15}^5 + 0.4253x_{15}^4 - 0.1347x_{15}^3 + 0.0277x_{15}^2 - 0.0033x_{15} + 0.0002) \quad \text{(with error of 1.58\%)} \tag{182}$$

$$y_{17} = 10^3(0.5847x_{15}^5 - 1.9282x_{15}^4 + 2.4383x_{15}^3 - 1.4744x_{15}^2 + 0.4273x_{15} - 0.0470 \quad \text{(with error of 9.82\%)} \tag{183}$$

$$y_{18} = 10^9(0.0035x_{15}^9 - 0.0153x_{15}^8 + 0.0270x_{15}^7 - 0.0238x_{15}^6 + 0.0088x_{15}^5 + 0.0018x_{15}^4 - 0.0033x_{15}^3 + 0.0014x_{15}^2 - 0.0003x_{15}) \quad \text{(with error of 5.94\%)} \tag{184}$$

$$y_{19} = 10^{10}(0.1522x_{15}^{10} - 0.8872x_{15}^9 + 2.2963x_{15}^8 - 3.4771x_{15}^7 + 3.4136x_{15}^6 - 2.2720x_{15}^5 + 1.0389x_{15}^4 - 0.3225x_{15}^3 + 0.0650x_{15}^2 - 0.0077x_{15} + 0.0004) \quad \text{(with error of 5.07\%)} \tag{185}$$

$$y_{20} = 10^{10}(-0.2555x_{15}^{10} + 1.4817x_{15}^9 - 3.8128x_{15}^8 + 5.7373x_{15}^7 - 5.5953x_{15}^6 + 3.6981x_{15}^5 - 1.6786x_{15}^4 + 0.5170x_{15}^3 - 0.1035x_{15}^2 + 0.0122x_{15} - 0.0006) \quad \text{(with error of 2.09\%)} \tag{186}$$

$$y_{21} = 10^{10}(-0.2567x_{15}^{10} + 1.5127x_{15}^9 - 3.9596x_{15}^8 + 6.0669x_{15}^7 - 6.0293x_{15}^6 + 4.0635x_{15}^5 - 1.8818x_{15}^4 + 0.5916x_{15}^3 - 0.1209x_{15}^2 + 0.0145x_{15} - 0.0008) \quad \text{(with error of 0.01\%)} \tag{187}$$

$$y_{22} = 10^{10}(-0.2953x_{15}^{10} + 1.7384x_{15}^9 - 4.5462x_{15}^8 + 6.9585x_{15}^7 - 6.9079x_{15}^6 + 4.6502x_{15}^5 - 2.1508x_{15}^4 + 0.6753x_{15}^3 - 0.1378x_{15}^2 + 0.0165x_{15} - 0.0009) \quad \text{(with error of 0.21\%)} \tag{188}$$

$$y_{23} = 10^{10}(0.1010x_{15}^{10} - 0.5867x_{15}^9 + 1.5109x_{15}^8 - 2.2730x_{15}^7 + 2.2133x_{15}^6 - 1.4584x_{15}^5 + 0.6589x_{15}^4 - 0.2016x_{15}^3 + 0.0400x_{15}^2 - 0.0046x_{15} + 0.0002) \quad \text{(with error of 0.36\%)} \tag{189}$$

$$y_{17} = 10^8(-0.1401x_{16}^7 + 0.5233x_{16}^7 - 0.8181x_{16}^6 + 0.7014x_{16}^5 - 0.3619x_{16}^4 + 0.1154x_{16}^3 - 0.0223x_{16}^2 + 0.0024x_{16} - 0.0001) \quad \text{(with error of 3.32\%)} \tag{190}$$

$$y_{18} = 10^{13}(0.1995x_{16}^{10} - 0.8723x_{16}^9 + 1.6601x_{16}^8 - 1.8169x_{16}^7 + 1.2704x_{16}^6 - 0.5944x_{16}^5 + 0.1889x_{16}^4 - 0.0403x_{16}^3 + 0.0055x_{16}^2 - 0.0004x_{16}) \quad \text{(with error of 2.34\%)} \tag{191}$$

$$y_{19} = 10^{10}(-0.5164x_{16}^9 + 2.0842x_{16}^8 - 3.5957x_{16}^7 + 3.4926x_{16}^6 - 2.1124x_{16}^5 + 0.8275x_{16}^4 - 0.2104x_{16}^3 + 0.0336x_{16}^2 - 0.0030x_{16} + 0.0001) \quad \text{(with error of 12.92\%)} \tag{192}$$

$$y_{20} = 10^{13}(0.1205x_{16}^{10} - 0.5262x_{16}^9 + 1.0003x_{16}^8 - 1.0934x_{16}^7 + 0.7634x_{16}^6 - 0.3567x_{16}^5 + 0.1132x_{16}^4 - 0.0241x_{16}^3 + 0.0033x_{16}^2 - 0.0003x_{16}) \quad \text{(with error of 3.53\%)} \tag{193}$$

$$y_{21} = -10^6(0.1387x_{16}^7 - 0.4505x_{16}^6 + 0.5903x_{16}^5 - 0.4065x_{16}^4 + 0.1603x_{16}^3 - 0.0366x_{16}^2 + 0.0045x_{16} - 0.0002) \quad \text{(with error of 14.65\%)} \tag{194}$$

$$y_{22} = -10^6(0.1036x_{16}^7 - 0.3376x_{16}^6 + 0.4442x_{16}^5 - 0.3078x_{16}^4 + 0.1225x_{16}^3 - 0.0283x_{16}^2 + 0.0035x_{16} - 0.0002) \quad \text{(with error of 12.53\%)} \tag{195}$$

$$y_{23} = 10^{13}(-0.3142x_{16}^{10} + 1.3749x_{16}^9 - 2.6186x_{16}^8 + 2.8682x_{16}^7 - 2.0071x_{16}^6 + 0.9400x_{16}^5 - 0.2990x_{16}^4 + 0.0639x_{16}^3 - 0.0088x_{16}^2 + 0.0007x_{16}) \quad \text{(with error of 20.76\%)} \tag{196}$$

$$y_{18} = 10^{11}(0.0104x_{17}^{10} - 0.0770x_{17}^9 + 0.2542x_{17}^8 - 0.4948x_{17}^7 + 0.6290x_{17}^6 - 0.5456x_{17}^5 + 0.3270x_{17}^4 - 0.1338x_{17}^3 + 0.0357x_{17}^2 - 0.0056x_{17} + 0.0004) \quad \text{(with error of 5.25\%)} \tag{197}$$

$$y_{19} = 10^{11}(0.1410x_{17}^{10} - 1.0658x_{17}^{9} + 3.6105x_{17}^{8} - 7.2204x_{17}^{7} + 9.4403x_{17}^{6} - 8.4318x_{17}^{5} + 5.2103x_{17}^{4} - 2.1996x_{17}^{3} + 0.6071x_{17}^{2} - 0.0989x_{17} + 0.0072) \quad \text{(with error of 10.31\%)} \tag{198}$$

$$y_{20} = 10^{5}(0.0704x_{17}^{6} - 0.3188x_{17}^{5} + 0.5916x_{17}^{4} - 0.5754x_{17}^{3} + 0.3087x_{17}^{2} - 0.0865x_{17} + 0.0099 \quad \text{(with error of 15.13\%)} \tag{199}$$

$$y_{21} = 10^{11}(-0.0213x_{17}^{10} + 0.1640x_{17}^{9} - 0.5670x_{17}^{8} + 1.1578x_{17}^{7} - 1.5465x_{17}^{6} + 1.4120x_{17}^{5} - 0.8924x_{17}^{4} + 0.3855x_{17}^{3} - 0.1090x_{17}^{2} + 0.0182x_{17} - 0.0014) \quad \text{(with error of 0.33\%)} \tag{200}$$

$$y_{22} = 10^{11}(-0.0226x_{17}^{10} + 0.1741x_{17}^{9} - 0.6020x_{17}^{8} + 1.2295x_{17}^{7} - 1.6428x_{17}^{6} + 1.5004x_{17}^{5} - 0.9487x_{17}^{4} + 0.4100x_{17}^{3} - 0.1159x_{17}^{2} + 0.0194x_{17} - 0.0015) \quad \text{(with error of 0.25\%)} \tag{201}$$

$$y_{23} = 10^{11}(-0.0825x_{17}^{10} + 0.6196x_{17}^{9} - 2.0869x_{17}^{8} + 4.1488x_{17}^{7} - 5.3916x_{17}^{6} + 4.7862x_{17}^{5} - 2.9392x_{17}^{4} + 1.2330x_{17}^{3} - 0.3382x_{17}^{2} + 0.0548x_{17} - 0.0040) \quad \text{(with error of 1.66\%)} \tag{202}$$

$$y_{19} = 10^{12}(-0.0116x_{18}^{10} + 0.0828x_{18}^{9} - 0.2637x_{18}^{8} + 0.4957x_{18}^{7} - 0.6086x_{18}^{6} + 0.5101x_{18}^{5} - 0.2956x_{18}^{4} + 0.1169x_{18}^{3} - 0.0302x_{18}^{2} + 0.0046x_{18} - 0.0003) \quad \text{(with error of 7.92\%)} \tag{203}$$

$$y_{20} = 10^{12}(-0.0242x_{18}^{10} + 0.1694x_{18}^{9} - 0.5313x_{18}^{8} + 0.9829x_{18}^{7} - 1.1881x_{18}^{6} + 0.9805x_{18}^{5} - 0.5594x_{18}^{4} + 0.2179x_{18}^{3}183 - 0.0555x_{18}^{2} + 0.0083x_{18} - 0.0006) \quad \text{(with error of 7.60\%)} \tag{204}$$

$$y_{21} = 10^{12}(0.1750x_{18}^{10} - 1.2310x_{18}^{9} + 3.8788x_{18}^{8} - 7.2105x_{18}^{7} + 8.7578x_{18}^{6} - 7.2621x_{18}^{5} + 4.1635x_{18}^{4} - 1.6296x_{18}^{3} + 0.4167x_{18}^{2} - 0.0629x_{18} + 0.0042) \quad \text{(with error of 25.32\%)} \tag{205}$$

$$y_{22} = -0.3013x_{18}^{2} - 0.2155x_{18} + 0.4963 \quad \text{(with error of 22.50\%)} \tag{206}$$

$$y_{23} = 10^{12}(0.0337x_{18}^{10} - 0.2372x_{18}^{9} + 0.7491x_{18}^{8} - 1.3957x_{18}^{7} + 1.6990x_{18}^{6} - 1.4119x_{18}^{5} + 0.8113x_{18}^{4} - 0.3182x_{18}^{3} + 0.0815x_{18}^{2} - 0.0123x_{18} + 0.0008) \quad \text{(with error of 7.44\%)} \tag{207}$$

$$y_{20} = 10^{12}(-0.0093x_{19}^{10} + 0.0544x_{19}^{9} - 0.1412x_{19}^{8} + 0.2151x_{19}^{7} - 0.2127x_{19}^{6} + 0.1429x_{19}^{5} - 0.0660x_{19}^{4} + 0.0207x_{19}^{3} - 0.0042x_{19}^{2} + 0.0005x_{19}) \quad \text{(with error of 3.95\%)} \tag{208}$$

$$y_{21} = 10^{12}(-0.0291x_{19}^{10} + 0.1661x_{19}^{9} - 0.4217x_{19}^{8} + 0.6271x_{19}^{7} - 0.6053x_{19}^{6} + 0.3965x_{19}^{5} - 0.1785x_{19}^{4} + 0.0546x_{19}^{3} - 0.0109x_{19}^{2} + 0.0013x_{19} - 0.0001) \quad \text{(with error of 18.68\%)} \tag{209}$$

$$y_{22} = 10^{12}(-0.0286x_{19}^{10} + 0.1630x_{19}^{9} - 0.4134x_{19}^{8} + 0.6142x_{19}^{7} - 0.5921x_{19}^{6} + 0.3874x_{19}^{5} - 0.1742x_{19}^{4} + 0.0532x_{19}^{3} - 0.0106x_{19}^{2} + 0.0012x_{19} - 0.0001) \quad \text{(with error of 14.59\%)} \tag{210}$$

$$y_{23} = 10^{12}(0.0844x_{19}^{10} - 0.4936x_{19}^{9} + 1.2853x_{19}^{8} - 1.9624x_{19}^{7} + 1.9462x_{19}^{6} - 1.3106x_{19}^{5} + 0.6070x_{19}^{4} - 0.1910x_{19}^{3} + 0.0391x_{19}^{2} - 0.0047x_{19} + 0.0003) \quad \text{(with error of 12.52\%)} \tag{211}$$

$$y_{21} = 10^{15}(0.0003x_{20}^{10} - 0.0023x_{20}^{9} + 0.0084x_{20}^{8} - 0.0182x_{20}^{7} + 0.0257x_{20}^{6} - 0.0248x_{20}^{5} + 0.0166x_{20}^{4} - 0.0076x_{20}^{3} + 0.0023x_{20}^{2} - 0.0004x_{20}) \quad \text{(with error of 17.86\%)} \tag{212}$$

$$y_{22} = 10^{10}(-0.1044x_{20}^{9} + 0.7624x_{20}^{8} - 2.4655x_{20}^{7} + 4.6340x_{20}^{6} - 5.5778x_{20}^{5} + 4.4585x_{20}^{4} - 2.3664x_{20}^{3} + 0.8042x_{20}^{2} - 0.1587x20 + 0.0139 \quad \text{(with error of 14.46\%)} \tag{213}$$

$$y_{23} = 10^{3}(0.2688x_{20}^{4} - 0.8261x_{20}^{3} + 0.9385x_{20}^{2} - 0.4660x_{20} + 0.0854 \quad \text{(with error of 23.79\%)} \tag{214}$$

$$y_{22} = 10^{13}(0.0771x_{21}^{10} - 0.1756x_{21}^{9} + 0.1420x_{21}^{8} - 0.0517x_{21}^{7} + 0.0090x_{21}^{6} - 0.0008x_{21}^{5}) \quad \text{(with error of 0.28\%)} \tag{215}$$

$$y_{23} = 10^{13}(0.5020x_{21}^{10} - 1.1458x_{21}^{9} + 0.9312x_{21}^{8} - 0.3424x_{21}^{7} + 0.0602x_{21}^{6} - 0.0055x_{21}^{5} + 0.0003x_{21}^{4}) \quad \text{(with error of 0.11\%)} \tag{216}$$

$$y_{23} = 10^{11}(3.2579x_{22}^{10} - 7.3439x_{22}^{9} + 5.9843x_{22}^{8} - 2.3215x_{22}^{7} + 0.4750x_{22}^{6} - 0.0556x_{22}^{5} + 0.0039x_{22}^{4} - 0.0002x_{22}^{3}) \quad \text{(with error of } 0.01\%) \tag{217}$$

where x_i is the measured results of test i; and y_i the estimated results of test i, with $3 \leq i \leq 23$. Details of test i can be seen in Table 1.

Appendix B. Summary of fitting errors (%) of 1st- to 10th-order polynomials

Equation	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8	Order 9	Order 10
(8)	3.65	3.42	3.16	2.88	2.68	2.08	1.21	1.30	0.88	12.81
(9)	0.42	0.41	0.41	0.14	0.13	0.12	0.13	0.12	0.18	1.53
(10)	2.37	2.25	2.07	2.00	1.85	1.38	0.74	0.60	2.38	3.21
(11)	1.05	1.01	1.03	0.88	0.94	0.94	0.94	0.93	0.42	2.74
(12)	34.66	32.66	31.93	30.96	30.08	24.26	8.49	5.12	4.23	2.18
(13)	17.37	17.62	17.47	15.77	12.15	8.52	8.25	2.61	11.10	2.13
(14)	17.81	19.78	17.73	20.11	19.38	18.85	12.56	11.87	−5.22	15.43
(15)	8.80	9.61	8.20	7.77	9.43	9.69	10.28	10.17	24.61	124.90
(16)	18.83	20.89	24.02	22.54	20.47	21.10	25.34	26.51	19.68	120.29
(17)	21.63	18.79	18.09	18.56	13.77	11.57	11.67	2.40	8.14	72.83
(18)	28.47	22.46	24.74	24.50	28.24	24.19	23.99	23.71	134.75	37.84
(19)	18.36	14.30	12.50	14.82	13.08	15.08	14.78	3.93	26.89	69.85
(20)	13.75	12.55	11.45	10.30	10.35	7.41	5.03	4.39	8.87	11.67
(21)	16.66	16.68	15.94	16.42	17.12	14.41	11.74	12.68	56.94	497.93
(22)	18.59	18.82	16.00	13.55	12.05	11.85	12.95	15.64	80.36	3.80
(23)	23.01	19.57	21.74	24.80	21.72	19.07	7.48	7.91	4.28	5.86
(24)	21.50	22.59	19.86	20.61	22.72	22.31	14.72	14.31	2.92	33.33
(25)	27.95	37.80	39.93	40.38	40.53	31.74	22.09	21.75	72.99	386.17
(26)	26.86	36.96	40.90	40.79	39.49	29.72	19.72	19.25	13.52	184.38
(27)	30.27	27.10	29.23	35.38	35.23	34.79	40.38	21.93	37.59	402.62
(28)	1.19	1.13	1.20	1.08	1.17	1.03	1.01	0.75	0.98	3.26
(29)	0.71	0.71	0.69	0.72	0.57	0.45	0.43	0.46	0.42	0.34
(30)	1.22	1.22	1.21	1.06	0.91	0.90	0.91	0.87	1.50	1.67
(31)	32.48	29.52	29.23	28.86	21.27	13.61	13.50	5.07	39.53	1.21
(32)	15.89	16.28	16.84	16.78	12.85	12.70	11.02	9.90	9.95	11.47
(33)	28.26	22.16	20.41	19.07	16.01	14.42	11.69	10.86	22.16	12.32
(34)	13.55	12.11	11.60	11.72	8.69	3.57	3.64	4.68	4.61	5.37
(35)	26.13	21.68	24.13	24.90	21.96	17.14	16.52	9.28	11.88	23.83
(36)	19.29	15.51	15.84	16.15	13.64	15.71	8.44	8.67	20.02	12.12
(37)	26.52	21.54	22.04	19.26	22.52	15.80	15.65	7.02	3.88	8.31
(38)	16.76	11.48	11.45	12.46	14.23	15.33	13.04	12.98	28.99	21.08
(39)	13.54	13.51	11.05	9.81	9.13	9.10	8.46	8.38	9.34	6.11
(40)	15.19	17.40	16.42	18.41	17.08	18.75	12.18	11.96	40.87	12.57
(41)	19.91	18.07	13.68	14.53	14.33	15.81	13.67	14.05	34.08	21.50
(42)	23.66	23.21	14.16	12.71	18.59	11.04	10.53	6.99	3.87	8.84
(43)	22.00	21.97	18.56	20.53	21.18	18.99	12.21	12.16	11.26	15.45
(44)	25.63	29.27	30.89	37.88	43.92	38.81	37.19	15.56	15.86	100.53
(45)	24.57	28.76	31.66	37.57	41.98	38.93	37.10	16.09	77.13	41.39
(46)	28.61	29.48	29.54	31.00	33.80	25.29	31.92	31.90	52.87	31.24
(47)	2.50	2.31	2.04	1.93	1.70	0.89	0.73	0.86	−0.08	15.25
(48)	0.94	0.99	1.03	0.73	0.81	0.67	0.67	0.48	0.31	146.88
(49)	34.70	32.49	31.38	30.91	29.10	17.24	8.41	8.86	−1.38	745.69
(50)	19.30	19.46	19.52	17.67	12.00	10.24	2.69	4.10	6.92	2184.94
(51)	19.67	22.20	18.23	20.70	19.74	18.66	11.84	23.92	5.91	3199.85
(52)	10.50	11.88	10.03	8.09	9.02	9.50	9.48	9.31	8.39	5830.37
(53)	19.98	23.91	25.93	21.85	20.65	20.92	25.29	26.62	13.20	8805.33
(54)	20.35	17.81	17.03	17.31	13.16	8.07	7.50	24.02	20.65	1203.30
(55)	27.06	22.59	25.45	25.55	28.23	26.06	25.14	27.08	37.36	85.14
(56)	18.95	14.07	11.55	14.63	12.11	13.92	8.65	28.50	24.01	75.57
(57)	12.74	12.71	13.02	12.49	12.20	11.54	6.34	19.75	16.96	296.14
(58)	16.66	16.19	17.90	17.48	19.78	16.95	12.59	38.78	23.35	16.92
(59)	18.08	18.35	16.88	14.77	12.11	12.81	12.78	19.33	31.15	860.46
(60)	23.37	20.01	22.42	24.85	20.29	13.76	8.38	8.03	2.65	4587.50
(61)	21.17	22.30	19.14	19.61	21.95	22.97	21.40	20.78	19.77	282.29
(62)	27.47	33.91	33.52	28.34	27.72	34.81	29.97	12.57	18.79	3538.93
(63)	26.42	33.12	34.81	29.48	27.73	34.81	30.14	12.18	23.48	14.91

Appendix B (*Continued*)

Equation	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8	Order 9	Order 10
(64)	30.58	26.70	29.61	37.07	35.66	34.71	36.35	13.13	26.84	2970.16
(65)	1.13	1.12	1.12	0.77	0.83	0.91	0.73	1.58	1.11	10303.00
(66)	32.54	29.88	29.50	29.45	23.75	11.03	2.99	1.45	1.72	2898.50
(67)	15.51	15.86	16.60	16.55	11.97	12.87	8.53	12.89	16.49	6191.19
(68)	27.64	21.55	19.73	19.28	16.36	13.94	6.84	11.35	12.59	227181.10
(69)	12.92	10.95	10.42	10.34	9.74	11.74	9.97	10.61	6.29	131291.99
(70)	25.27	23.74	26.49	21.72	22.26	23.46	16.97	26.92	18.97	467111.48
(71)	19.58	15.19	15.13	14.53	16.64	17.33	18.80	21.53	21.20	28269.94
(72)	27.13	22.53	21.49	18.11	13.02	15.04	14.13	13.76	15.39	210389.44
(73)	19.03	12.35	12.69	13.05	13.27	14.44	10.54	15.78	21.76	103963.07
(74)	13.18	13.61	13.21	8.60	8.15	6.08	6.77	5.52	6.77	108765.85
(75)	15.67	16.11	14.92	19.70	21.70	22.86	24.29	13.39	12.22	220733.08
(76)	19.99	17.95	12.94	14.22	12.98	13.27	12.78	24.50	26.85	230322.96
(77)	24.21	24.07	13.51	14.63	15.63	7.01	7.09	8.53	8.06	121682.13
(78)	21.90	22.01	18.27	20.12	20.99	15.88	14.45	8.43	3.32	364700.00
(79)	25.78	31.45	33.97	39.09	49.65	48.70	49.16	50.06	47.69	1926352.97
(80)	24.75	30.83	34.57	39.13	48.54	48.61	49.04	49.14	52.74	991998.45
(81)	28.44	29.33	29.85	32.40	37.75	31.61	40.31	2.07	3.36	1145545.16
(82)	34.62	31.69	30.29	28.25	24.12	22.58	22.05	6.59	3139.70	362.87
(83)	34.21	31.08	32.72	31.89	30.98	29.78	32.51	13.61	−96.31	166391.19
(84)	46.70	49.89	54.59	54.64	53.35	45.14	45.76	69.58	4533.07	3875.59
(85)	38.64	43.16	45.76	46.22	45.23	38.63	35.06	16.46	642.87	55908.64
(86)	47.22	51.01	54.76	55.08	53.56	47.80	45.71	23.03	9780.33	342463.11
(87)	34.01	28.03	31.69	30.37	29.56	21.35	26.29	34.59	78.30	323137.42
(88)	34.66	38.04	41.84	43.54	42.99	32.28	36.67	30.28	1639.44	32731.69
(89)	24.04	29.57	28.74	22.70	19.24	15.73	20.49	20.17	1363.07	46736.96
(90)	23.58	25.12	26.29	22.68	22.87	29.53	29.66	53.87	290.85	79980.69
(91)	18.45	14.08	10.58	9.58	10.06	11.35	11.76	50.64	2571.95	217500.00
(92)	18.05	18.87	19.97	22.20	19.01	15.13	16.16	25.44	829.32	358335.10
(93)	28.93	19.12	14.77	13.48	8.79	7.90	7.37	7.79	5167.60	195232.40
(94)	15.95	17.72	21.24	18.78	19.08	15.86	13.02	4.47	1316.67	134316.67
(95)	29.81	27.74	34.72	31.57	36.73	29.91	61.15	161.27	6388.93	9000.00
(96)	28.20	26.65	33.53	30.66	34.43	27.19	58.88	144.52	3397.89	4647.89
(97)	35.12	31.05	30.50	23.28	23.47	24.67	1.25	3.28	1495.16	1990.32
(98)	60.33	38.77	34.28	30.93	29.32	20.94	15.16	40.42	−75.89	43.08
(99)	58.63	41.36	37.34	32.75	29.90	11.82	8.74	27.39	588.47	301.30
(100)	64.31	44.10	38.65	34.11	17.38	9.74	9.88	17.97	−67.60	5546.14
(101)	56.54	44.36	38.90	34.07	30.21	10.90	3.68	13.72	41.57	600.64
(102)	55.08	37.82	34.23	33.13	24.04	15.42	20.05	52.10	−86.29	88.04
(103)	59.86	37.00	34.69	33.43	19.47	24.90	24.23	27.34	−185.21	8833.10
(104)	19.65	28.43	35.45	25.42	6.46	6.40	7.45	20.10	−28.34	2770.00
(105)	23.45	28.47	36.79	28.39	12.20	14.54	12.22	11.99	401.58	852.90
(106)	18.40	18.17	15.76	17.26	20.62	20.65	9.55	5.99	552.83	1896.94
(107)	21.80	19.25	17.94	17.19	13.51	16.82	18.38	53.70	−57.31	8791.71
(108)	14.88	18.46	20.77	20.23	22.96	23.05	22.58	19.27	24.40	4149.67
(109)	23.20	18.57	22.20	20.84	20.75	19.91	19.69	20.11	−42.19	2300.00
(110)	18.90	31.07	31.57	33.18	33.14	28.99	13.22	13.34	2355.02	2561.07
(111)	18.12	28.74	29.36	32.36	33.95	29.28	11.37	9.47	1515.98	2335.09
(112)	25.61	28.85	27.95	26.67	30.75	41.24	41.25	10.24	1633.06	2629.84
(113)	19.16	20.96	18.15	20.01	11.92	7.75	8.59	9.95	8.61	1.87
(114)	12.45	12.80	10.84	9.45	7.76	6.36	5.05	5.23	5.38	7.18
(115)	19.98	22.09	24.74	23.02	17.98	20.94	18.92	19.39	20.15	2.42
(116)	27.09	21.03	18.38	16.18	11.44	6.63	5.30	3.01	0.64	0.23
(117)	33.04	25.63	29.93	29.57	28.39	24.88	25.29	24.99	25.38	15.84
(118)	20.98	12.49	12.17	8.19	6.95	6.68	6.16	6.77	6.64	2.44
(119)	16.08	13.61	18.11	8.44	8.19	8.12	8.10	9.05	8.08	7.44
(120)	14.71	14.49	13.16	17.96	19.04	19.47	18.68	18.15	16.97	6.34
(121)	22.68	18.31	18.52	11.21	10.69	13.41	12.50	9.36	8.87	8.15
(122)	23.34	18.91	19.92	22.48	13.45	7.57	7.23	7.69	4.21	5.27
(123)	21.90	23.07	20.54	22.06	23.57	20.17	19.29	17.64	16.73	18.53
(124)	31.49	43.86	46.83	47.77	34.20	30.22	27.76	28.86	30.43	24.94
(125)	30.05	42.43	46.72	47.65	34.85	27.81	24.62	25.49	26.91	20.73
(126)	27.11	31.50	30.29	43.02	34.53	31.04	26.32	31.73	33.01	32.58
(127)	18.04	14.23	14.96	17.10	15.01	15.89	15.85	6.76	6.76	9.89

Appendix B (*Continued*)

Equation	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8	Order 9	Order 10
(128)	14.29	13.14	13.88	11.49	11.11	11.96	10.90	9.22	0.82	27.32
(129)	15.96	15.60	16.36	16.64	17.80	16.26	16.32	15.34	10.43	279.56
(130)	32.20	15.55	18.70	19.33	18.29	18.32	15.92	19.16	14.09	2920.53
(131)	25.74	7.71	10.19	9.00	8.98	6.86	5.42	6.41	4.53	1194.78
(132)	73.47	73.84	74.11	76.27	75.46	74.79	73.84	76.80	79.87	60.38
(133)	15.18	15.07	16.12	15.52	16.95	17.91	14.67	9.19	8.88	1191.67
(134)	22.36	16.45	16.69	11.48	11.48	11.48	11.48	11.48	11.48	142.52
(135)	26.88	22.07	22.83	22.81	22.81	22.81	22.81	22.81	22.81	41.34
(136)	21.75	20.62	18.32	17.38	16.67	16.67	16.67	16.67	16.67	432.81
(137)	23.19	23.44	22.89	23.96	23.89	34.17	30.42	41.68	49.03	1863.93
(138)	22.12	22.36	21.78	20.29	21.84	31.68	30.73	41.47	48.94	2127.33
(139)	24.48	25.21	30.20	20.17	22.97	10.55	8.58	10.57	12.90	459.12
(140)	16.12	16.66	18.11	18.11	18.67	19.33	17.21	13.28	12.07	3.86
(141)	16.38	16.25	15.87	12.26	10.27	11.82	11.84	10.49	10.32	11.77
(142)	22.75	21.40	21.46	20.82	20.81	19.49	20.90	17.46	19.83	14.17
(143)	16.26	11.84	10.86	11.04	10.47	10.86	11.83	12.38	11.30	9.54
(144)	13.13	13.98	9.19	11.01	6.81	6.09	6.10	5.38	5.46	6.25
(145)	14.44	15.77	14.71	14.64	16.82	8.59	11.13	11.61	8.12	8.25
(146)	19.04	18.74	15.14	15.11	15.77	15.26	12.03	11.90	7.99	6.57
(147)	21.92	20.48	22.15	21.59	10.59	7.17	7.04	4.99	2.81	0.95
(148)	22.93	21.65	20.80	22.12	14.82	15.12	12.71	8.08	5.33	5.29
(149)	24.40	25.13	25.07	22.24	29.79	40.48	34.37	37.31	39.59	36.41
(150)	23.32	22.15	22.11	21.74	26.90	39.26	34.47	36.67	39.49	36.46
(151)	27.84	36.34	36.26	30.97	40.13	36.17	37.23	36.31	35.10	21.83
(152)	18.52	14.56	15.35	17.17	13.24	12.76	10.43	10.43	10.43	10.52
(153)	29.01	20.41	19.99	18.97	17.98	17.76	16.71	14.94	14.08	7.41
(154)	24.89	11.82	10.08	13.08	6.92	6.93	6.63	4.07	4.09	4.23
(155)	17.02	12.36	11.92	11.85	11.86	11.86	11.86	11.86	11.86	11.99
(156)	11.86	15.12	10.51	12.76	11.95	11.95	8.88	8.88	8.88	38.18
(157)	22.27	17.69	19.14	17.59	15.20	11.48	11.48	11.48	11.48	11.55
(158)	26.01	22.66	22.88	22.81	22.82	22.81	22.81	22.81	22.81	22.73
(159)	21.69	17.86	17.06	16.66	16.85	16.67	16.67	16.67	16.67	16.74
(160)	25.06	26.33	40.89	40.26	45.49	44.71	46.30	47.36	49.33	1123.31
(161)	23.28	25.07	38.81	39.14	44.75	44.24	46.23	47.31	48.95	1125.07
(162)	24.23	33.16	24.98	27.50	27.89	28.35	23.46	21.85	12.90	241.10
(163)	25.83	26.82	29.46	28.29	26.41	25.81	17.44	12.52	10.51	3.52
(164)	20.61	12.78	12.80	7.22	6.69	6.64	3.98	3.32	4.08	4.24
(165)	24.30	19.46	21.03	11.39	11.19	10.16	9.58	9.57	6.66	8.55
(166)	16.33	15.53	16.23	18.43	18.03	19.47	12.95	10.54	10.60	12.23
(167)	22.53	17.76	19.80	15.08	14.60	14.88	13.76	6.96	1.39	0.28
(168)	24.65	21.04	20.79	23.32	19.05	7.19	6.15	7.50	6.75	5.47
(169)	22.71	23.10	19.22	19.92	21.86	18.76	18.56	18.79	13.75	12.50
(170)	22.93	26.89	25.20	30.23	25.62	33.51	35.05	43.08	34.75	1.79
(171)	20.96	25.27	24.33	29.12	24.54	33.48	35.09	42.47	32.43	2.33
(172)	26.91	30.77	30.32	36.67	28.63	26.07	26.42	29.81	4.88	1.61
(173)	17.28	10.48	16.92	15.71	15.63	16.13	10.00	10.61	0.65	17.29
(174)	26.53	17.59	17.02	10.89	11.02	7.50	7.07	5.06	3.43	0.17
(175)	25.38	19.03	27.44	21.79	22.16	19.97	12.44	11.78	11.74	456.70
(176)	20.30	18.50	15.01	20.72	20.82	17.23	14.75	14.17	9.57	173.38
(177)	23.55	21.76	25.94	21.81	19.57	12.11	12.49	9.36	9.38	1.81
(178)	22.55	21.23	20.20	20.21	22.82	17.06	17.21	17.59	17.59	36.24
(179)	80.99	35.73	64.04	52.73	51.03	50.59	35.25	1.76	0.41	8.31
(180)	80.81	32.97	64.81	55.04	48.81	50.09	34.18	1.02	0.37	0.94
(181)	59.32	31.41	58.80	59.75	59.77	43.57	38.18	42.60	0.00	0.00
(182)	12.40	16.20	6.56	10.69	10.60	11.66	9.78	9.67	2.64	1.58
(183)	16.35	18.61	14.81	14.44	9.82	10.28	11.28	11.14	11.22	10.09
(184)	14.21	13.49	15.61	15.53	12.25	14.06	13.94	5.98	5.94	8.35
(185)	19.04	23.26	12.07	12.37	10.83	10.69	9.38	8.83	9.54	5.07
(186)	20.67	21.18	18.32	18.22	20.58	17.54	18.73	13.86	10.72	2.09
(187)	24.68	22.81	25.09	31.58	38.33	31.82	42.27	20.75	11.08	0.01
(188)	21.91	20.86	23.53	29.14	36.68	31.02	41.04	22.51	12.78	0.21
(189)	29.30	34.49	33.34	34.47	36.52	39.02	39.00	3.53	3.33	0.36
(190)	14.78	16.65	18.45	20.98	20.28	18.47	10.51	3.32	4.00	5.46
(191)	18.45	17.87	18.15	18.88	18.16	19.59	18.45	12.62	9.38	2.34

Appendix B (*Continued*)

Equation	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8	Order 9	Order 10
(192)	22.43	23.21	17.68	19.18	18.13	18.16	17.73	14.73	12.92	17.15
(193)	23.93	23.24	19.67	15.91	19.01	18.99	16.84	17.04	8.43	3.53
(194)	24.57	32.11	33.78	27.82	16.92	15.72	14.65	14.79	20.60	24.90
(195)	23.56	32.05	33.70	23.58	14.18	13.34	12.53	12.60	18.00	22.52
(196)	26.33	24.86	28.20	32.46	33.71	31.87	30.30	39.71	32.40	20.76
(197)	23.87	22.72	22.28	19.77	14.07	14.33	13.90	9.98	6.64	5.25
(198)	24.52	25.68	23.70	19.19	22.18	25.44	26.27	23.71	24.34	10.31
(199)	25.26	19.68	19.47	16.13	16.13	15.13	19.34	19.73	16.17	15.96
(200)	33.80	44.08	52.47	43.93	19.73	24.98	27.06	17.14	4.70	0.33
(201)	30.94	41.20	49.87	41.55	21.64	28.22	30.20	18.35	5.00	0.25
(202)	27.84	29.97	30.96	25.51	25.36	25.97	35.18	31.13	18.62	1.66
(203)	24.82	27.23	26.68	28.12	29.90	32.51	19.37	12.76	9.46	7.92
(204)	25.24	21.48	22.91	22.03	22.90	19.33	22.22	18.96	11.38	7.60
(205)	28.72	26.04	34.34	35.05	37.09	38.76	39.99	45.86	42.80	25.32
(206)	23.94	22.50	31.24	31.79	35.08	36.77	38.44	44.32	42.00	25.14
(207)	25.74	19.07	16.96	17.10	14.03	20.46	12.22	18.06	9.99	7.44
(208)	18.19	17.08	16.90	14.44	13.98	10.22	8.30	5.17	5.06	3.95
(209)	21.62	23.28	23.38	28.32	28.96	38.03	41.23	42.15	19.37	18.68
(210)	20.90	23.32	23.04	29.61	29.94	36.54	42.16	43.02	16.90	14.59
(211)	31.28	31.85	33.49	39.13	36.74	43.95	43.67	45.97	37.70	12.52
(212)	21.17	26.59	29.54	34.41	38.04	33.48	36.63	19.65	17.98	17.86
(213)	20.45	26.81	26.52	34.49	37.92	33.77	36.92	19.15	14.46	14.74
(214)	33.17	33.42	29.91	23.79	27.00	28.32	23.95	35.90	37.10	43.50
(215)	6.71	2.83	2.77	1.54	1.61	1.73	1.92	2.44	0.74	0.28
(216)	24.17	18.97	25.61	24.58	28.01	27.69	8.66	3.98	4.84	0.11
(217)	23.08	20.77	24.06	23.38	20.34	23.86	20.64	12.74	5.37	6.82E−05

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